Household Borrowing Constraints and Residential Investment Dynamics

Hashmat Khan*  
Carleton University

Jean-François Rouillard†  
Université de Sherbrooke

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Abstract

Why does residential investment lead output in the US and Canada but it is coincident in eurozone countries? In this paper we explore the role of home-equity loans used to boost consumption as a channel that affects residential investment. We consider a multi-agent model where some home-owning households face borrowing constraints that reflect home-equity loans or refinancing constraints. The main contribution of our paper is to highlight that the severity of the households’ borrowing constraints in an economy can generate both stylized facts of residential investment dynamics. In US and Canada, a greater proportion of households rely on home-equity loans relative to eurozone countries. This difference matters for the distinct residential investment dynamics observed across countries.

Key words: Home-Equity Loans, Borrowing Constraints, Residential Investment, Business Cycles

JEL classification: E22, E32, R21, R31

*Corresponding Author. Department of Economics, D891 Loeb, 1125 Colonel By Drive, Carleton University, Ottawa, Canada. E-mail: hashmat.khan@carleton.ca. Tel: +1.613.520.2600 (ext 1561).

†Department of Economics, Université de Sherbrooke, Sherbrooke, Canada. E-mail: j-f.rouillard@usherbrooke.ca.
Of the components of GDP, residential investment offers by far the best early warning sign of an oncoming recession - Leamer (2008)

1 Introduction

Residential investment in the housing sector has long been viewed as important for understanding fluctuations in economic activity (Burns and Mitchell (1946)). Recently, Leamer (2008, 2015) has noted that the decline of cumulative residential investment in recessions accounts for about half of the overall decline in US GDP. In this context, two key stylized facts have been documented in the literature: First, residential investment leads the business cycle in US and Canada. Second, residential investment is coincident with the cycle in eurozone countries.\(^1\) Kydland et al. (2014) present a model with long-term mortgage loans and multi-period time-to-build in residential construction to account for these stylized facts. They model the loan structure as ‘first mortgage’ loans and, therefore, do not capture an important source of financing household consumption, namely, home-equity or refinancing loans. Gorea and Midrigan (2015) have pointed out that mortgage refinancing accounts for about one-third of the rise and fall in household spending during the 2001-2011 period in the US.

In this paper we explicitly explore the role of home-equity loans to refinance consumption. We consider a multi-agent model with homeownership, where some home-owners face borrowing constraints that reflect home equity loans or refinancing constraints. To model this scenario we use the Iacoviello (2005) framework where homeowners facing borrowing constraints have a smaller discount factor relative to those who do not.

The main contribution of our paper is to highlight that the severity of household borrowing constraints in the economy can generate both stylized facts of residential investment dynamics as described above. The model, therefore, provides an endogenous explanation for the two stylized facts.

We show that residential investment leads output by one quarter when the fraction of house-

\(^1\) Fisher (2007) studies a separate stylized fact of why might residential investment lead non-residential investment over the business cycle in the US. He emphasizes the complementarity of household capital—as proxied by the size of house—to labour and capital in market production in accounting for this fact. However, in Fisher’s model residential investment is coincident with output.
holds who face borrowing constraints is matched to US data. In eurozone countries, the share of households facing borrowing constraints is, on average, relatively smaller than the US, which according to the model implies that residential investment is coincident with the cycle. One piece of corroborative evidence at the aggregate level is high household-debt-to-GDP ratio in the US relative to the eurozone. From data provided by the Bank for International Settlements, Canadian and American average household debt over the 1999 to 2015 time-period corresponds to 77.64 and 84.05 percent of their annualized GDPs, whereas for the eurozone the same ratio is 56.35 percent. Moreover, according to the IMF (2011), in 2004-05, the share of households that had a mortgage was 20 percent in the eurozone and 45 percent in the US. We base our model calibration on this evidence.

There are two incentives for purchasing larger quantity of houses. First, housing services directly enter the households’ utility function. Since the positive technology shock induces a wealth effect, they decide to spend more on houses. However, this positive wealth effect alone is not sufficient to produce the leading pattern. Second, a share of households are borrowing-constrained. The accumulation of the housing stock, therefore, allows them to borrow more contemporaneously and in future periods, so that they are also able to consume greater quantities of the non-durable good. The household borrowing constraint and its effect on residential investment is, therefore, important in understanding the two prominent stylized facts mentioned above.

Our paper is related to Ren and Yuan (2014) who attempt to explain why residential investment leads the cycle in the US, using a partial equilibrium model with collateral constraints, agent heterogeneity, and Total Factor Productivity (TFP) news shocks. However, their model has the odd implication that agents prefer to purchase houses instead of consumption goods after a positive endowment shock. This is necessarily the case in their model because the mortgage interest rate is lower than the credit card rate, both of which are calibrated parameters. In actual economies, even though the mortgage rate is lower than the credit card rate, the use of credit card for consumer goods purchases is common place. Moreover, their model cannot explain the stylized facts for eurozone countries where residential investment is coincident with the cycle.2

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2 Another aspect that is ignored in the partial equilibrium setting of Ren and Yuan (2014) is that they do not have the labour supply decision in the model. This assumption is not innocuous because TFP news shocks, which are essential to their explanation, can produce wealth effects that mitigate business cycle
The rest of the paper is organized as follows. Section 2 presents the model. Section 3 describes the calibration of parameters. Section 4 presents the quantitative results. Section 5 presents sensitivity analysis. Finally, section 6 concludes.

2 The model

The model economy is populated by four categories of agents: impatient and patient households, entrepreneurs and savers. Agents are infinitely-lived and maximize a discounted sum of time-separable utilities. We assume that impatient households and entrepreneurs borrow from savers, therefore their discount factors, $\theta_I$ and $\theta_E$, are lower than the savers’ discount factor, $\beta$. The impatient households’ loans are collateralized by the expected value of their stock of housing. Both types of households are employed by firms that produce non-durable goods and houses — a durable good. They derive some utility from the consumption of non-durable goods and housing services. Entrepreneurs own both types of firms and make non-residential and residential investments. They also face a borrowing constraint, as the value of their debt cannot exceed a fraction of their capital stock. Finally, we assume that savers are risk-neutral agents whose only purpose is to channel funds to the two other categories of agents. Since their share in the economy is not important for the dynamics, we do not allocate any share for these agents. The shares in the economy of the impatient and patient households, and entrepreneurs are respectively $\omega_I$, $\omega_P$, and $\omega_E$. In the appendix, we present a fully detailed description of the model’s stationary version.

2.1 Households

The impatient households’ maximization problem is as follows:

$$\max E_0 \sum_{t=0}^{\infty} \theta_I^t \left( \ln c_{It} + \psi \ln h_{It} + \eta_I \ln \left( 1 - (n_{It}^{1+\epsilon} + n_{Iht}^{1+\epsilon})^{1/(1+\epsilon)} \right) \right)$$

subject to

$$c_{It} + (1 + \tau_h)q_{ht}h_{It} - q_{ht}(1 - \delta_h)h_{It-1} = b_{It} - R_{t-1}^{e}b_{Ht-1} + (1 - \tau_n) \sum_{i=c,h} w_{it}n_{It} + \xi_{It},$$

$$b_{It} \leq m_H E_t q_{ht+1}h_t.$$ (3)
First, note that variables $c_{It}$, $h_{It}$, $n_{Ict}$, $n_{Iht}$, $b_{It}$, and $\xi_{It}$ are variables that are expressed per impatient household. Their period utility function is in logarithms and weighs their consumption of the non-durable good, $c_{It}$, their housing stock, $h_{It}$, and their leisure. The specification for leisure is not standard, as we assume that the labor force is split up between the production of non-durable goods, $n_{Ict}$, and house production, $n_{Iht}$. We follow Horvath (2000) and Iacoviello and Neri (2010) and allow for an imperfect substitution of labor between sectors that is governed by parameter $\epsilon$. A greater value of $\epsilon$ implies a lower reallocation of labor between sectors that is governed by parameter $\epsilon$.

On the right-hand-side of their budget constraint, equation (2), impatient households have different sources of revenues: (i) they earn labor income that is taxed at rate $\tau_n$, so that their total after-tax labor income is $(1 - \tau_n) \sum_{i=c,h} w_{it} n_{Iit}$ where wages $w_{it}$ are taken as given, (ii) they borrow $b_{It}$ at the effective interest rate $R^e_t = 1 + r_t (1 - \tau_m)$ where $r_t$ is the interest rate and $\tau_m$ corresponds to the fraction of deductible interest payments. We consider adjustable rate mortgages, as they are renegotiated every period.

The second constraint, equation (3), is a collateral constraint. Specifically, the level of debt contracted every period cannot exceed a fraction $m_H$ of the expected value of the households’ stock of housing, i.e. $E_t q_{ht+1} h_{It}$. Since impatient households have a lower discount factor, this constraint is binding in the steady state. We consider small shocks and a sufficiently low discount factor so that it is also always binding in the neighborhood of the steady state.

The patient households’ problem departs from the impatient households’ problem as their higher discount factor makes it suboptimal for them to carry debt ($\beta > \theta_I$). However, their utility function takes the same form as is shown by the following equations:
max \( E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln c_P + \psi \ln h_P + \eta \ln \left( 1 - \left(n_{Pct}^{1+\epsilon} + n_{Pht}^{1+\epsilon}\right)^{1/(1+\epsilon)} \right) \right) \) (4) subject to 
\[ c_P + (1 + \tau_h)q_{ht} - q_{ht} (1 - \delta_h) = (1 - \tau_n) \sum_{t=c,h} w_{it} n_{pit} + \xi_{Pit} \] (5)

Note that the variables with subscript \( P \) are the counterparts for patient households of variables described below for the impatient households. All variables are expressed per patient household.

### 2.2 Entrepreneurs

The entrepreneurs maximize the following problem:

\[
\max E_0 \sum_{t=0}^{\infty} \theta_E^t \ln c_{Et} \] (6)

subject to

\[ c_{Et} + x_{ct} + k_{bt} = b_{Kt} - R_{t-1} b_{Kt-1} + [1 - \tau_k] \left( r_{ct} k_{ct-1} + p_{bt} k_{bt} \right) + p_{lt} l_t + \xi_{Et}, \]

\[ x_{ct} = k_{ct} - (1 - \delta_c) k_{ct-1}, \]

\[ b_{Kt} \leq m_K k_{ct}. \] (9)

Note that variables \( c_{Et}, x_{ct}, b_{Kt}, k_{ct}, l_t, k_{bt}, \) and \( \xi_{Et} \) are variables that are expressed per entrepreneur. The variable \( c_{Et} \) corresponds to the entrepreneurs’ consumption of non-durable goods. They also invest in the capital used for the production of these goods, \( x_{ct} \), and in the materials that are used in the construction of houses, \( k_{bt} \). The law of motion of capital, \( k_{ct} \), is given by equation (8). In contrast, there is no accumulation of materials. The revenues of the entrepreneurs are shown in the right-hand-side of equation (7). First, firms pay a rental rate \( r_{ct} \) to use capital, \( k_{ct-1} \). Second, entrepreneurs sell \( k_{bt} \), to the firms at price \( p_{bt} \). Their capital revenue which consists in the proceeds of the sale of capital and materials is taxed at rate \( \tau_k \). Third, entrepreneurs are endowed with \( l_t \) acreage in land every period that they sell to firms at price \( p_{lt} \). Fourth, since they have a lower discount factor, they borrow \( b_{Kt} \) at the interest rate \( R_t \), so that their net borrowing is

\[ \text{For simplicity, we assume that they do not work nor do they own houses.} \]
Fifth, they receive governmental transfers $\xi E_t$. Similar to impatient households, entrepreneurs face a borrowing constraint, equation (9), that states that they cannot borrow more than a fixed fraction $m_K$ of their capital stock.

### 2.3 Savers

Savers are risk-neutral unconstrained agents who lend to impatient households and entrepreneurs. Since their discount factor is greater than discount factors of the two other categories of agents, they are savers in the economy. Their maximization problem is as follows:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t c_{St} \quad \text{(10)}$$
subject to

$$c_{St} = \omega_I (R_{t-1} b_{Ht-1} - b_{Ht}) + \omega_E (R_{t-1} b_{Kt-1} - b_{Kt}). \quad \text{(11)}$$

where $c_{St}$ corresponds to the consumption of a saver. Since we assume that categories of agents can have different shares in the total population, the total amount lent to impatient households and entrepreneurs each period corresponds to $\omega_I b_{Ht}$ and $\omega_E b_{Kt}$.

### 2.4 Firms and production

Firms operate in a perfectly competitive market to produce non-durable goods, $y_t$, and new houses, $ih_t$. Their problem is static and is as follows:

$$\max y_t + q_{ht} ih_t - \sum_{i=c,h} \omega_{it} \sum_{j=I,P} \omega_j n_{jit} - \omega_{E} r_{ct} k_{ct-1} - \omega_{E} (p_{lt} l_t + p_{kt} k_t)$$
subject to

$$y_t = z_t \left( \omega_{E} k_{ct-1} \right)^{\alpha_C} \left( \sum_{j=I,P} \omega_j n_{jct} \right)^{1-\alpha_C}$$

$$ih_t = z_t \left( \omega_{E} k_{bt} \right)^{\alpha_B} \left( \omega_{E} l_t \right)^{\alpha_L} \left( \sum_{j=I,P} \omega_j n_{jht} \right)^{1-\alpha_B-\alpha_L}.$$

The variables $y_t$ and $ih_t$ are variables expressed per capita. Every period, the firms pay factors of production from the sale of non-durable goods and houses. The labor inputs, capital stock, and
land have to be adjusted to account for the difference in population shares of the three categories of agents. The assumption of competitive markets ensures that profit is null in equilibrium. We assume that hours worked by the impatient and patient households are perfectly substitutable inputs for the production of the two categories of goods. We also assume that both production functions are Cobb-Douglas and that neutral technology, \( z_t \), enters both of them. This shock follows an AR(1) process:

\[
\ln z_t = \rho \ln z_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2)
\]  

(12)

where \( \rho \) corresponds to the persistence parameter and \( \sigma^2 \) to the variance of the innovation.

### 2.5 Government

In this economy, the government does not play any productive role. It levies distortionary taxes on households, entrepreneurs, and firms, and makes lump-sum transfers, so that the total value of transfers corresponds exactly to the amount that these two categories of agents pay in taxes:

\[
\xi_{It} = \omega_I \left( \tau_n \sum_{i=c,h} w_{it} n_{It} + \tau_h q_{ht} h_{It} - \tau_m r_{t-1} b_{Ht-1} \right),
\]  

(13)

\[
\xi_{Pt} = \omega_P \left( \tau_n \sum_{i=c,h} w_{it} n_{Pt} + \tau_h q_{ht} h_{Pt} \right),
\]  

(14)

\[
\xi_{Et} = \omega_E \left[ r_{ct} k_{ct-1} + p_{kt} k_{kt} \right].
\]  

(15)

### 2.6 Market clearing conditions

There is a resource constraint in the economy, so that output corresponds to the sum of consumption and investment:

\[
y_t = \sum_{i=I,P,E} \omega_i c_{it} + c_{St} + \omega_E \left( x_{ct} + k_{kt} \right).
\]  

(16)

Additionally, the law of motion for houses is as follows:

\[
i_h t = h_t - (1 - \delta_H) h_{t-1}
\]  

(17)

where \( h_t = \omega_I h_{It} + \omega_P h_{Pt} \). We assume that land supply is equal to one for all periods.\(^4\)

\(^4\)In similar fashion to Davis and Heathcote (2005), we do not model land dynamics over the business cycle.
3 Calibration

Table 1 shows the calibration of the baseline model. A study by the IMF (2011) approximates the share of households with a mortgage to be 45 percent in 2004-05 for the US. In the economy, we choose to have the same share of households that are borrowing-constrained (impatient), i.e. $\omega_I = 0.45$. From the Survey of Consumer Finances (SCF), Cagetti and De Nardi (2006) report that the percentage of business owners in the population is 9.1 percent, therefore we set $\omega_E = 0.09$.

The share of patient households corresponds to the remaining share, i.e. $\omega_P = 0.46$. The value of the patient households and savers’ quarterly discount factor is set so that the real interest rate is 3 percent in the steady state. Since impatient households and entrepreneurs are borrowing-constrained, they face a shadow cost of borrowing in addition to interest payments. This cost can be considered as an interest premium. Similar to Bernanke, Gertler and Gilchrist (1999), we choose the impatient households’ discount factor to match a two percent interest premium, i.e. $\theta_W = \theta_E = 0.973$. For a reasonable size of the TFP shock, this value of the discount factor ensures that the borrowing constraint is always binding.

The weight on housing in the households’ utility function is governed by $\psi$ and is set to match the average household debt-to-GDP-ratio in the US from 1984 to 2014, i.e. 2.89 from data provided by the Bank for International Settlements. The weights assigned to leisure, $\eta_I$ and $\eta_P$, are picked so that households work 30 percent of their allocated time in the steady state. The parameter that governs the degree of sectoral labor mobility, $\epsilon$, is the only one that is determined outside the steady state. It is set to match the relative volatility of residential investment to GDP. From the 2001 SCF data, Gorea and Midrigan (2015) find a mean loan-to-value from all mortgage holders of 0.52, therefore we set $m_H = 0.52$. As for the entrepreneurs’ loan-to-value, $m_K$, we pick a value so that the baseline model in the steady state generates the same ratio of non-financial businesses debt securities and loans liabilities to GDP from 1984 to 2014, i.e. 2.47.

Taxation distorts the decisions of households and entrepreneurs. For labor and capital taxes, $\tau_n$ and $\tau_k$, we take the values estimated by Gomme and Rupert (2007) who use Mendoza et al. (1994) methodology to calculate effective average tax rates. We set the property tax rate, $\tau_h = 0.00625$, that is the quarterly value reported for the US by Girouard et al. (2006) in their survey of housing.
TABLE 1: Calibration of the baseline model

**Population**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_I$</td>
<td>0.45</td>
<td>fraction of impatient households</td>
</tr>
<tr>
<td>$\omega_P$</td>
<td>0.46</td>
<td>fraction of patient households</td>
</tr>
<tr>
<td>$\omega_E$</td>
<td>0.09</td>
<td>fraction of entrepreneurs</td>
</tr>
</tbody>
</table>

**Discount factors, preferences and loan-to-values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9925</td>
<td>discount factor (savers and patient households)</td>
</tr>
<tr>
<td>$\theta_W$</td>
<td>0.973</td>
<td>discount factor (impatient households)</td>
</tr>
<tr>
<td>$\theta_E$</td>
<td>0.973</td>
<td>discount factor (entrepreneurs)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.455</td>
<td>weight on housing</td>
</tr>
<tr>
<td>$\eta_I$</td>
<td>1.921</td>
<td>weight on leisure (impatient households)</td>
</tr>
<tr>
<td>$\eta_P$</td>
<td>2.007</td>
<td>weight on leisure (patient households)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.21</td>
<td>sectoral labor mobility</td>
</tr>
<tr>
<td>$m_H$</td>
<td>0.52</td>
<td>imp. households' loan-to-value</td>
</tr>
<tr>
<td>$m_K$</td>
<td>0.426</td>
<td>entrepreneurs’ loan-to-value</td>
</tr>
</tbody>
</table>

**Taxation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_n$</td>
<td>0.22</td>
<td>labor income tax</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>0.29</td>
<td>capital tax</td>
</tr>
<tr>
<td>$\tau_h$</td>
<td>0.00625</td>
<td>property tax</td>
</tr>
<tr>
<td>$\tau_m$</td>
<td>0.4</td>
<td>fraction of deductible mortgage interest payments</td>
</tr>
</tbody>
</table>

**Depreciation, and technology**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_m$</td>
<td>0.025</td>
<td>depreciation of capital</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>0.004</td>
<td>depreciation of housing</td>
</tr>
<tr>
<td>$\alpha_C$</td>
<td>0.36</td>
<td>elasticity with respect to capital (non-durable goods)</td>
</tr>
<tr>
<td>$\alpha_B$</td>
<td>0.53</td>
<td>elasticity with respect to material inputs (housing)</td>
</tr>
<tr>
<td>$\alpha_L$</td>
<td>0.1</td>
<td>elasticity with respect to land (housing)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.95</td>
<td>TFP persistence</td>
</tr>
</tbody>
</table>

9
taxation across OECD countries. Mortgage interest payments can be deducted from taxable income in the US. We also follow Girouard et al. (2006) work, so that the after-income tax mortgage rate, $\tau_m$, corresponds to 60% of the real interest rate’s value.

The depreciation rate of capital is standard, i.e. $\delta_m = 0.025$. We follow Gomme and Rupert (2007) in computing the housing depreciation rate from BEA data and set $\delta_h = 0.004$. We set the capital share in the production of houses, $\alpha_C = 0.36$. We follow Davis and Heathcote (2005) and Iacoviello and Neri (2010) and set the share of land in the production of new houses to $\alpha_L = 0.1$. In the production function of houses, we do not distinguish between capital in structures and intermediate goods. Yet material costs compose a large share of total house production costs. Specifically, in the US input-output tables from 1997 to 2014, this share is 15% greater than workers’ compensation. We set $\alpha_H$ to match this ratio to which we add a 10% share for structures in similar fashion to the two studies aforementioned. Finally, we choose the persistence parameter of the TFP shock, $\rho = 0.95$.

4 Quantitative results

In this section, we present the results for two different calibrations of the baseline model: one that matches moments of the US data (values presented in Table 2) and another one for which we lower the share of households that have mortgage debt (impatient households) to match data from eurozone. A first set of results is that the presence of borrowing constraints is critical to replicate the lead in residential investment and many co-movements and relative volatilities for the US that a standard real business cycle model fails to deliver. Second, we show that when we lower the share of impatient households in the economy to the share that prevailed in the eurozone, the lead in residential investment vanishes.

4.1 Benchmark calibration

In Table 2 we report the relative volatilities and co-movements of consumption, residential investment, non-residential investment, hours worked in construction and in other sectors, and housing prices. We have set the degree of sectoral labor mobility measured by $\epsilon$ to match the relative standard deviation of residential investment, so we have imposed the model to match this moment.
TABLE 2: Baseline Model Properties

<table>
<thead>
<tr>
<th>STATISTIC</th>
<th>U.S. DATA</th>
<th>BENCHMARK CALIBRATION</th>
<th>EUROZONE MORTGAGE DEBT $(\omega_I = 0.2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(c_t, y_t)$</td>
<td>0.89</td>
<td>0.86</td>
<td>0.88</td>
</tr>
<tr>
<td>$\rho(k_{bt}, y_t)$</td>
<td>0.6</td>
<td>0.7</td>
<td>0.91</td>
</tr>
<tr>
<td>$\rho(x_{ct}, y_t)$</td>
<td>0.85</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>$\rho(n_{ht}, y_t)$</td>
<td>0.8</td>
<td>0.72</td>
<td>0.92</td>
</tr>
<tr>
<td>$\rho(n_{ct}, y_t)$</td>
<td>0.8</td>
<td>0.51</td>
<td>0.76</td>
</tr>
<tr>
<td>$\rho(i_{ht}, y_t)$</td>
<td>0.59</td>
<td>0.73</td>
<td>0.92</td>
</tr>
<tr>
<td>$\rho(q_{ht}, y_t)$</td>
<td>0.61</td>
<td>0.35</td>
<td>0.72</td>
</tr>
</tbody>
</table>

**Co-movements**

**Relative volatilities**

| $\sigma_c / \sigma_y$ | 0.87 | 0.48 | 0.44 |
| $\sigma_{kb} / \sigma_y$ | 6.74 | 6.72 | 5.82 |
| $\sigma_{xc} / \sigma_y$ | 4.13 | 3.97 | 4.00 |
| $\sigma_{kb} / \sigma_{xc}$ | 1.63 | 1.69 | 1.46 |
| $\sigma_{nc} / \sigma_y$ | 1.05 | 0.77 | 0.43 |
| $\sigma_{nh} / \sigma_y$ | 5.25 | 4.79 | 4.25 |
| $\sigma_{ih} / \sigma_y$ | 10.93 | 6.17 | 5.45 |
| $\sigma_{gh} / \sigma_y$ | 3.54 | 0.61 | 0.41 |

Notes: The U.S. data spans from 1984Q1 to 2015Q1 (except for hours worked series that start in 1985Q1). All series are logged and detrended with the HP-filter $(\lambda=1600)$. Data sources are in the appendix. In the last two columns, $c$ corresponds to total consumption, i.e. $c = \sum_{i=L,P,E} \omega_i c_i + c_S$, and $n_c$ and $n_h$ correspond to total hours worked in the consumption sector and in the housing sector, i.e. $n_c = \omega_I n_{Ic} + \omega_P n_{Pc}$ and $n_h = \omega_I n_{Ph} + \omega_P n_{Ph}$. 


However, there are no restrictions on the values of all other moments. The model reproduces very well the relative volatilities of non-residential investment and sectoral hours worked. Specifically, the volatility of investment is significantly higher in the housing construction sector both in the data and generated by the model. This success is shared with previous work, e.g. Davis and Heathcote (2005), Kydland et al. (2014), and Fisher (2007), however, a standard multi-sector real business model as in Greenwood and Hercowitz (1991) fails at replicating this relative volatility. The effects of borrowing constraints are not at work here — it is simply the consequence of a lower depreciation rate of houses than capital used in the production of non-durable goods. In fact, the effects of investing in housing production last longer with a lower depreciation rate.

Moreover, the hours worked in the housing production sector are more volatile than hours worked in the production of non-durable goods. This result is simply due to the greater volatility of the production of houses itself, which in return is related to durability of this category of goods. Accounting for the high volatility of housing prices in the data remains a challenge for our model. In the literature, it appears that technology shocks are not sufficient to explain the radical shifts in housing prices. Finally, the model is able to replicate the positive co-movements in all aggregate quantity variables as well as in housing prices.

In Figure 1, we present the cross-correlation functions between output and residential investment computed from the actual data and those generated by the benchmark model. Residential investment leads output by one quarter. The one period lead generated by the model overshoots its data counterpart, but the predictions for the remaining leads and lags are consistent with the data.

The impulse responses displayed in Figure 2 gives evidence of the borrowing constraint mechanism at play. There are two incentives for purchasing larger quantity of houses. First, housing services directly enter the households’ utility function. Since the positive technology shock induces a wealth effect, they decide to spend more on houses. However, this positive wealth effect alone is not sufficient to produce the leading pattern. The key characteristic is that a fraction of these households are borrowing-constrained. Therefore, the accumulation of the housing stock allows them to borrow more contemporaneously and in future periods, so that they are also able to consume greater quantities of the non-durable good.
Notes: Specifically, this function corresponds to the correlation between residential investment at time $t + j$ and output at time $t$. The blue line corresponds to the correlations generated by the benchmark model and the red line to the correlations estimated from US data between 1984Q1 and 2015Q1. All series are logged and detrended with the HP-filter ($\lambda=1600$).

The increase in residential investment after a technology shock is much stronger than the increase in non-residential investment, since the demand for houses is greater than for non-durable goods. Because the entrepreneurs’ borrowing constraint is also relaxed from the accumulation of capital, however, the initial response of non-residential investment is remains positive. In subsequent periods, its response follows the one of output which explains the strong contemporaneous correlation. As for hours worked, the reallocation from the non-durable sector to the housing sector is stronger as a wage differential arises.

4.2 Eurozone calibration for household borrowing

After having stressed the role played by borrowing constraints for the lead of residential investment for the US, we calibrate the financial frictions parameters to match the data of eurozone countries.
Figure 2: Impulse responses to a one percent technology shock.

Notes: In panel a, the dashed blue line corresponds to the response of residential investment and the solid red line to non-residential investment. In panel b, the solid green line corresponds to output.

As seen in the introduction, there is a high degree of heterogeneity across countries when we examine mortgage-debt-to-GDP ratios. According to the IMF (2011), in 2004-05, the share of households that had a mortgage was 20 percent in the eurozone. This smaller share is the only parameter that we re-calibrate relative to the benchmark calibration, i.e. $\omega_I = 0.2$. 
Figure 3: Impulse responses to a one percent technology shock: Eurozone calibration

Notes: In panel a, the dashed blue line corresponds to the response of residential investment and the solid red line to non-residential investment. In panel b, the solid green line corresponds to output.

We report the results in the fourth column of Table 2. Most relative volatilities are not affected by the change in calibration. The two exceptions are the volatilities of the price of houses and the hours worked for the production of non-durable goods. The incentives to build more houses are lessened, simply because less households are facing borrowing constraints for which their houses are used as collateral. Since the demand for houses is not as strong, the volatility of its relative price is lower. For the same reason, the substitution of sectoral labor is reduced and non-residential investment increases following the technology shock as shown in Figure 3. Since the technology shock has similar effects for the production of non-durable goods and houses, the aggregate quantity variables are strongly correlated. As a consequence, the lead in residential investment vanishes as can be seen in Figure 4. This result is consistent with the cross-correlation pattern for France, indicating a very good fit of the model.

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\[^5\]We choose to report the cross-correlation pattern for France for two reasons. First, it is the second-largest
Notes: Specifically, this function corresponds to the correlation between residential investment at time $t + j$ and output at time $t$. The blue line corresponds to the correlations generated by the benchmark model and the red line to the correlations estimated from French data between 1984Q1 and 2015Q1. All series are logged and detrended with the HP-filter ($\lambda=1600$).

5 Sensitivity analysis

In the previous section we showed that changing the value of the share of impatient households in the economy greatly affects the leading pattern. Figure 5 presents the difference between the correlation of residential investment at time $t - 1$ and output at time $t$ and the correlation of these two variables at time $t$ for a wide range of the shares of impatient households ($\omega_I$) in the economy. All the other parameters remain the same as reported in Table 1. A positive gap would indicate that the absolute magnitude of the leading correlation is greater than contemporaneous correlation. The gap between the leading and contemporaneous correlations is unambiguously a economy in the eurozone. Second, its statistical agency reports a long time span of residential investment data. Moreover, from 1984 to 2015, its household-debt-to-GDP ratio is approximately half of the US's (53.63 percent to be precise).
positive function of the fraction of borrowing-constrained households. This result sheds light on the importance of the households’ borrowing collateral channel.

6 Conclusion

Residential investment leads the business cycle in the US and Canada. In eurozone countries, however, residential investment is coincident with the output. The main contribution of this paper is to show that home-equity loans used for consumption can influence residential investment dynamics in ways that can account for both stylized facts. We show that when a larger proportion of households face borrowing constraints on home-equity loans, as in the US and Canada, residential investment can lead the business cycle. When this proportion is relatively smaller, as in eurozone countries, residential investment becomes coincident with the output and no longer serves as a leading indicator in the economy.
References


Gorea, D. and Midrigan, V.: 2015, Liquidity constraints in the us housing market.


A Data

A.1 US

The series for output, consumption, residential investment, and non-residential investment are from NIPA, Table 1.1.3. New houses are from the US Bureau of Census, specifically this series “Housing Starts: Total: New Privately Owned Housing Units Started”. The hours worked series are from the BLS, $n_{ht}$ is the number of employees in the residential construction sector, whereas $n_{ct}$ consists in the difference between total employment and $n_{ht}$. The housing price index is constructed from FRB data, following the decomposition proposed by Davis (2009), which is divided by the CPI index from the BLS.

A.2 France

Data is from the Quarterly National Accounts published by the INSEE.

B The model equations

B.1 Impatient households

\[
\max E_0 \sum_{t=0}^{\infty} \theta^t_1 \left( \ln c_{It} + \psi \ln h_{It} + \eta_I \ln \left( 1 - \left( n_{Ict}^{1+\epsilon} + n_{Iht}^{1+\epsilon} \right)^{1/(1+\epsilon)} \right) \right)
\]

subject to

\[
c_{It} + (1 + \tau_h)q_{ht}h_{It} - q_{ht}(1 - \delta_h)h_{I-1} = b_{Ht} - R_{t-1}^{e}\left( \frac{b_{Ht}}{1 - \tau_n} \sum_{i=c,h} w_{it} n_{Iit} + \xi_{lt} \right),
\]

\[
b_{Ht} \leq m_{I} E_{lt} q_{ht+1} h_{lt},
\]

\[
n_t = \left( n_{Ict}^{1+\epsilon} + n_{Iht}^{1+\epsilon} \right)^{1/(1+\epsilon)},
\]

\[
R_t^e = 1 + r_t(1 - \tau_m).
\]

First-order conditions

\[ c_{It}: \]

\[
\frac{1}{c_{It}} = \lambda_{It}
\]

\[ b_{Ht}: \]

\[
\lambda_{It} - \lambda_{2t} = \theta_I R_t^e E_t \lambda_{It+1}
\]

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\( h_{lt}: \)
\[
q_{lt} \lambda_{lt}(1 + \tau_l) - n_l^H \lambda_{lt} E_t q_{lt+1} = \theta_l(1 - \delta_l) E_{lt+1} q_{lt+1} + \frac{\Psi}{h_{lt}}
\]  
(25)

\( n_{ljt} \quad j = c, h: \)
\[
\lambda_{lt}(1 - \tau_n) w_{jt} = \frac{\eta_l}{1 - n_l} \left( \frac{n_{ljt}}{n_{lt}} \right)^\epsilon
\]  
(26)

B.2 Patient households

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln c_{lt} + \psi \ln h_{lt} + \eta_P \ln \left( 1 - (n_{Pt}^{1+\epsilon} + n_{Pht}^{1+\epsilon})^{1/(1+\epsilon)} \right) \right)
\]
subject to
\[
c_{lt} + (1 + \tau_h) q_{lt} h_{lt} - q_{lt}(1 - \delta_l) h_{lt-1} = (1 - \tau_n) \sum_{i=c,h} w_{it} n_{Plt} + \xi_{lt},
\]  
(28)
\[
n_t = (n_{Pc}^{1+\epsilon} + n_{Pht}^{1+\epsilon})^{1/(1+\epsilon)}.
\]  
(29)

First-order conditions

\( c_{lt}: \)
\[
\frac{1}{c_{lt}} = \lambda_{lt}
\]  
(30)

\( h_{lt}: \)
\[
q_{lt} \lambda_{lt}(1 + \tau_h) = \beta(1 - \delta_l) E_{lt} \lambda_{lt+1} q_{lt+1} + \frac{\Psi}{h_{Plt}}
\]  
(31)

\( n_{Pjt} \quad j = c, h: \)
\[
\lambda_{lt}(1 - \tau_n) w_{jt} = \frac{\eta_P}{1 - n_{Pjt}} \left( \frac{n_{Pjt}}{n_{Pjt}} \right)^\epsilon
\]  
(32)

B.3 Entrepreneurs

\[
\max E_0 \sum_{t=0}^{\infty} \theta_{Et}^t \ln c_{Et}
\]
subject to
\[
c_{Et} + x_{ct} + k_{lt} = (1 - \tau_k)[r_{ct}k_{ct-1} + p_{lt}k_{lt}] + b_{Kt} - R_{lt-1} b_{Kt-1} + p_{lt} + \xi_{Et},
\]  
(34)
\[
b_{Kt} \leq m_{Kt} k_{ct}
\]  
(35)
\[
x_{ct} = k_{ct} - (1 - \delta_c) k_{ct-1}.
\]  
(36)
First-order conditions

c_{Et}:
\[ \frac{1}{c_{Et}} = \mu_{1t} \] (37)

b_{Kt}:
\[ \mu_{1t} - \mu_{2t} = \theta_E R_t E_t \mu_{1t+1} \] (38)

k_{et}:
\[ \mu_{1t} \mu_{2t} = \theta_E E_t (\mu_{1t+1} (1 - \delta_c + (1 - \tau_k) r_{ct+1})) \] (39)

k_{bt}:
\[ \mu_{1t} = \mu_{1t} (1 - \tau_k) p_{bt} \] (40)

B.4 Savers

\[ \max \sum_{t=0}^{\infty} \beta^t c_{St} \] (41)
subject to
\[ c_{St} = \omega_I (R_{t-1} b_{Ht-1} - b_{Ht}) + \omega_E (R_{t-1} b_{Kt-1} - b_{Kt}) \] (42)

First-order conditions

c_{St}:
\[ 1 = \chi_{1t} \] (43)

b_{Ht}:
\[ 1 = \beta R_t \] (44)
B.5 Firms and production

\[
\max y_t + q_{ht}i_{ht} - \sum_{i=c,h} w_{it} \sum_{j=I,P} \omega_j n_{jit} - \omega_{ERct} k_{ct-1} - \omega_{EPlt} l_t
\]

subject to

\[
y_t = z_t (\omega_E k_{ct-1})^{\alpha_C} \left( \sum_{j=I,P} \omega_j n_{jct} \right)^{1-\alpha_C},
\]

\[
i_{ht} = z_t (\omega_E k_{bt})^{\alpha_B} (\omega_E l_t)^{\alpha_L} \left( \sum_{j=I,P} \omega_j n_{jht} \right)^{1-\alpha_B-\alpha_L}.
\]

Law of motion of houses:

\[
i_{ht} = h_t - (1 - \delta_h) h_{t-1}
\]

\[
h_t = \omega_I h_{lt} + \omega_B h_{lt}.
\]

First-order conditions

\(n_{jct}:
\]

\[
w_{ct} = (1 - \alpha_C) \frac{y_t}{\sum_{j=I,P} \omega_j n_{jct}}
\]

\(n_{jht}:
\]

\[
w_{ht} = (1 - \alpha_B - \alpha_L) \frac{q_{ht} h_{lt}}{\sum_{j=I,P} \omega_j n_{jht}}
\]

\(k_{ct-1}:
\]

\[
\omega_{ERct} = \alpha_C \frac{y_t}{k_{ct-1}}
\]

\(k_{bt}:
\]

\[
\omega_{EPbt} = \alpha_B \frac{q_{ht}}{k_{bt}}
\]

\(l_t:
\]

\[
\omega_{EPlt} = \alpha_L \frac{q_{ht}}{l_t}
\]

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