Evidence that Financial Markets are Efficient Sometimes and Inefficient Most of the Time

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Abstract

The issue of market efficiency attracted the attention of academicians since the existence of financial markets. Over time, two schools of thoughts were established: the efficient markets school and the behavioral finance school. Proponents of the former believed in the Efficient Markets Hypothesis whereas the latter brought evidence from psychology neurosciences to demonstrate the irrationality of investors in making financial decisions. Recently, an adaptive hypothesis was suggested. This paper proves mathematically the existence of this adaptability process and tests empirically its validity. The results support the adaptability process,−namely that markets are indeed efficient sometimes and inefficient most of the time.

Keywords: Efficient Markets Hypothesis, Behavioral Finance, Adaptive Markets Hypothesis, Smooth transition regression models, Threshold models, regime switching models, Financial Markets, Irrationality, S&P500, Market-to-Book ratio.

1 Introduction

Academicians have had interests in studying and understanding the behavior of financial time series data since the existence of financial markets. The

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analysis of the process of price formation in these markets was, and still is, a topic of special interest for both practitioners and academicians. Numerous efforts were devoted to studying this process and resulted in many equilibrium models of price formation. Unfortunately, these models, elegant as they were, did not succeed in capturing entirely the dynamic behavior of financial price series. Over time, however, two schools of thoughts were established: the efficient markets school and the behavioral finance school. Proponents of the former believed in the Efficient Markets Hypothesis (EMH), which posits that capital markets are ideal in the sense that, at any point in time, asset prices in any market fully reflect all available information. The advocates of the EMH were then set to develop equilibrium models of price formation that are implicitly based on the EMH, which has the presumption of investor rationality at its core.[1] The latter school, on the other hand, brought evidence from behavioral finance, psychology, and neurosciences showing that investors, especially retail traders, exhibit irrational behavior in making investment decisions, i.e., asset allocation and portfolio construction, which can explain the observed violations of the EMH in capital markets.

The process of asset allocation and portfolio construction is considered a building bock of modern finance theory, which has its routes in the late 1950s in the work of Harry Markowitz (1959), who studied the investor's portfolio decision problem. This problem, according to Markowitz, is to choose optimal weights of individual assets in a given portfolio such that the variance of the rate of return of the portfolio is minimized for a given expected return. Harry showed that the solution to this problem is a locus of risk and return combinations that yield the minimum portfolio variance for a given rate of return. Soon after Harry's contribution, a formal equilibrium model of price formation that explains the risk-reward relationship for individual assets or portfolios of assets in the financial markets was introduced. The

[1] For an excellent survey on the theoretical and empirical foundations of the EMH, see Fama (1970) and the references therein.

model, which is known as the Capital Asset Pricing Model (CAPM), was originally developed by Sharpe (1964) and further extended and clarified by Lintner (1965). A slightly different version was supplied by Fisher Black (1972). Although the CAPM, as a model of price formation, is intuitively appealing, its empirical validity is questionable.[2] In fact, when put to the test, the model's empirical performance was so poor to the extent that it was rendered invalid. But, despite the model's empirical failure, it remains the most favorable asset pricing model among practitioners in the finance industry. This is perhaps due to its simplicity and wide applicability.

In 1976, the economist Stephen Ross suggested another more general and more testable alternative to CAPM. Unlike CAPM, which suggests that the rate of return on any security is linearly related to a single factor, which is the rate of return on the market portfolio, Ross's theory, which is known as the Arbitrage Pricing Theory (APT), asserts that the variation in the rate of return of a security can be explained by its sensitivity to a number of factors. This last statement is a statement about factor models, which are considered the building block of the APT. The idea here is that the rate of return on a security depends on market variables and firm-specific variables. The former are economic variables that have influence on the economy, the market, and, in turn, the rate of return on the security under consideration, e.g., changes in interest rate, changes in inflation, technological changes, and business cycle fluctuations, whereas the latter are firm related shocks, known as idiosyncratic shocks, such as defective products, harmful products, bad publicity, reputation, and poor management.

Recent evidence shows that other factors than the stock market beta, which is suggested by CAPM, seem to have an explanatory power in explaining the variation of the expected rate of return on a stock. Basu (1977) discovered an earning-to-price (E/P) ratio effect. In particular, Basu showed

[2] For an excellent recent survey on the empirical tests of the validity of the CAPM, see Fama and French (2004) and the references therein.

that when common stocks are sorted on E/P, future returns on high E/P stocks are higher than predicted by CAPM. Banz (1981) reported a size effect; when stocks are sorted on market capitalization, average returns on small stocks are higher than predicted by CAPM. Bhandari (1988) reported a leverage effect; high debt-to-equity ratios are associated with returns that are too high relative to their market betas. Statman (1980) and Rosenberg, Reid, and Lanstein (1985) showed that stocks with high book-to-market equity ratios have high average returns than the returns suggested by their beta coefficients.

The evidence brought by the previous studies and many others confirms that the standard version and the black version of the CAPM suffer from fatal problems. The failure of CAPM as an equilibrium theory of price formation prompted researchers to turn to explanations. The equilibrium that I am referring to here is the financial equilibrium in capital markets, where the demand for financial assets is equal to the supply. In fact, Markowitz's model, CAPM, and even Ross's APT, assume that all asset prices clear the markets of all assets. These attempts, unfortunately, do not explain or derive this financial equilibrium; rather they assume that it is the result of market *efficiency*. Thus, the common underlying hypothesis of these models is the EMH.

In general, two explanations appear in the literature: one explanation of the failure of CAPM entertains the fact that the model is based on many unrealistic assumptions, and thus, a more complicated model of price formation is needed. Advancements on this frontier made use of various types of micro-founded models in explaining the behavior of investors and in deriving equilibrium conditions in all markets. (See, for instance, Campbell and Cochrane (1999) and Epstein and Zin (1989).) The other explanation is due to the behaviorists. They brought recent evidence from behavioral finance and neurosciences to demonstrate that the market is inefficient. Their rationale is that investors, especially retail traders, exhibit irrational behavior

in constructing portfolios and conducting their trades in the market. This irrational behavior causes asset prices to over or under value their fundamental values rendering the market inefficient. Andrew Lo (2004) surveyed the literature on the debate between the advocates of the EMH and behavioral finance and suggested reconciliation between both approaches. His reconciliation hypothesis, which he called "the Adaptive Markets Hypothesis (AMH)," posits that the inefficiency in the market is due to the irrational behavior of investors, but since investors adapt to the changing environment, their adaptability over time brings the market back to efficiency. Basically, Lo's AMH posits that financial markets witness episodes of inefficiencies, but the adaptability of its participants forces them to revert back to efficiency and so on. Putting it differently, one can argue that financial markets are efficient sometimes and inefficient most of the time. In this paper, we formally define this adaptability process and test its empirical validity.[3]

The previous discussion suggests that, conditional on a given information set, which we will denote by Ω in the text, investors make financial *choices* concerning asset allocations and portfolio construction. In other words, given Ω, investors choose the types of assets and their corresponding optimal weights in forming their portfolios such that a target rate of return $r_p := w_1 r_1 + w_2 r_2 + ... + w_n r_n$ is achieved, where r_i is the rate of return on asset i, w_i is the weight of asset i in portfolio p, for $i = 1, ..., n$. These choices, however, are subject to change according to changes, or updates, in the information set Ω; every time a bit of information/news is acquired, the information set Ω is updated. This causes investors to continuously alter the composition of their portfolios, i.e., changing the weights of the assets forming their portfolios by taking short or long positions in some or all of the assets. This, in turn, will alter the demand for these financial assets, which will alter their current trading prices causing their market value to be above or below the book value. Therefore, the previous discussion suggests the

[3] So, in a way, this paper can be seen as an empirical test of the AMH.

existence of a continuous map from Ω to the market-to-book value (M/B) of individual assets such that updates in Ω induces episodes of inefficiencies, where asset prices are undervalued or overvalued. Since any financial market is seen as a portfolio of assets and since the market rate of return is simply the weighted sum of these individual assets, then the previous proposition means that the market rate of return continuously switches from a lower inefficient regime where asset prices are undervalued to an upper inefficient regime where asset prices are overvalued, and in between these two inefficient regimes the market rate of return passes briefly by an episode of efficiency, where its market value is equal to its book value. The objective of this paper is to, first, prove mathematically the existence of the previous map from the information set Ω to the M/B of the individual assets forming the market portfolio, and second, to test empirically its validity.

The paper is organized as follows. Section 2 states and rigorously proves the previous proposition on the process of adaptability of investors' choices in capital markets. Section 3 provides the empirical analysis and evidence supporting the proposition. Finally, Section 4 concludes.

2 The Adaptability Process

The adaptability process in financial markets can be thought of as the continuous adjustment of portfolio weights by investors following continuous changes or updates in their information set Ω. In other words, the adaptability process can be described by a continuous map from Ω to $\mathbb{W}$, where $\mathbb{W}$ is the set of portfolio weights. In order to prove the existence of such function formally, one could put a general, but sensible, topological structure on its domain and codomain. But, before we proceed with a formal statement, we need to specify the nature of elements contained in the domain and codomain of such a function, i.e., the nature of elements in both sets Ω and $\mathbb{W}$.

Consider portfolio p, which consists of n assets and let $w_i \in \mathbb{W}$ be the

weight of asset i in portfolio p, for $i = 1, ..., n$, such that $\sum_{i=1}^{n} w_i = 1$ and $0 < w_i \leq 1$. Thus, $\mathbb{W}$ could be taken to be the half open interval $(0, 1]$. As for the information set, we will use the digital media time as a proxy of information. Digital media time is the time investors spent on financial news sites, rating sites, or other social media, to learn about financial assets, read financial news, or gather information about a particular asset or group of assets. Let τ_{it} denote the digital time spent on acquiring news about asset i at time t, for $i = 1, ..., n$. We will be interested in studying the relation between any *two* consecutive digital times, i.e., two successive visits to a particular news site, τ_{it} and its lag τ_{it-1}. Therefore, we will be studying pairs of digital times $(\tau_{it-l}, \tau_{it-l-1})$ for $i = 1, ..., n$, $l = 0, 1, .., k$, where k is the distance in time between the visit made at time $t - l$ and that made at time $t-l-k$. Hence, the information $\Omega = \mathbb{R}^{2n+} \backslash \{0\}$ is a subset of $\mathbb{R}^{2n}$, whose n elements, 2-tuples each, are strictly positive real digital times $(\tau_{it-l}, \tau_{it-l-1})$ for $l = 0, 1, ..., k$ and $i = 1, ..., n$.

Note that the sequence $\{\tau_{it-l}\}_{l=0}^{k}$ can be thought of as the update process, where τ_{it-k} is the oldest visit to the site and τ_{it} is the most recent visit. So, for instance, if the update process is assumed to be non-increasing, then we get the following sequence of visits

$$\tau_{it} \leq \tau_{it-1} \leq \tau_{it-2} \leq ... \leq \tau_{it-k}.$$

This means that an investor devotes greater time when he first learns about asset i. This time could be taken to be some sort of fundamental analysis performed by the investor or his analyst when constructing the portfolio. If the assumption that further updates take fewer digital media times till the most recent update, τ_{it}, is entertained, then asset i could be thought of as a stable asset, e.g., a blue ship corporation in the stock market. Of course, there are many other possibilities to model the sequence of digital time and they indeed depend on the riskiness of the asset under consideration. We are not interested, however, in modelling the sequence of visits. Our interest is

rather to model the relationship between two consecutive digital times in the sequence.

A reasonable assumption to make is that the digital time devoted to asset i follows an autoregressive process of order 1, or AR(1) for short, as

$$\tau_{it-l} = \phi_i \tau_{it-l-1}, \text{ for } i = 1, ..., n,$$

where ϕ_i is the autocorrelation coefficient corresponding to asset i.[4] Notice here that since time can not be zero or take negative values, $\phi_i > 0$. Moreover, if we entertain the assumption that the time series τ_{it-l} is covariance stationary, then $0 < \phi_i < 1$. Due to the nature of financial news, it is sensible to assume that the time spent following it displays some sort of cycle, i.e., it can be above or below the trend, and hence, stationarity of the time series τ_{it-l} is a sensible presumption. Now, we are ready to give a formal proposition and prove it.

Proposition 1 *Let $w_i \in \mathbb{W} = (0, 1]$ be the weight of asset i in portfolio p, for $i = 1, ..., n$, such that $\sum_{i=1}^{n} w_i = 1$ and $0 < w_i \leq 1$. Let $\mathbb{R}^{2n+}\backslash\{0\}$, a subset of $\mathbb{R}^{2n}$, whose n elements, 2-tuples each, are strictly positive real digital times $(\tau_{it-l}, \tau_{it-l-1})$ for $l = 0, 1, ..., k$, and $i = 1, ..., n$. If the time series τ_{it-l} follows an AR(1) process with $\phi_i > 0$ as*

$$\tau_{it-l} = \phi_i \tau_{it-l-1}, \tag{1}$$

then, for every asset i, there exists a continuous function

$$\bar{f}_i : \Omega \longrightarrow \mathbb{R}^+, \ \bar{f}_i\left([(\tau_{it-l}, \tau_{it-l-1})]\right) = \left(\frac{\tau_{it-l}}{\tau_{it-l-1}}\right)^k,$$

where $\Omega := \mathbb{R}^{2n+}\backslash\{0\}/\sim$ is the information set, and the relation $\sim$ on $\mathbb{R}^{2+}\backslash\{0\}$ is such that for $x, y \in \mathbb{R}^{2+}\backslash\{0\}$, $x \sim y$ if $y = \phi x$, where $\phi > 0$.

[4] Different orders of autoregressive processes could also be assumed. However, to simplify the analysis, an AR(1) will suffice.

Furthermore, if τ_{it-l} is assumed to be covariance stationary, then the function $g : \Omega \longrightarrow \mathbb{W}$ defined as

$$g\left(\left[\left(\tau_{1t-l}, \tau_{1t-l-1}\right)\right], \left[\left(\tau_{2t-l}, \tau_{2t-l-1}\right)\right], ..., \left[\left(\tau_{nt-l}, \tau_{nt-l-1}\right)\right]\right) = \left(\phi_1^k, \phi_2^k, ..., \phi_n^k\right),$$

is a continuous function that maps pairs of digital times, as proxies of information updates on the n assets, from the information set Ω to the set of portfolio weights $\mathbb{W}$.

Proof. First note that, by recursive substitution, it is easy to see that, for asset i, the process in (1) can be expressed as

$$\tau_{it-l} = \phi_i^k \tau_{it-l-1}, \tag{2}$$

where k is the distance in time between the visit at time $t-l$ and that at time $t-l-k$. Equation (2) states that, for asset i, the difference between any two consecutive updates depends on the autoregressive coefficient ϕ_i and the distance in time k.

Now, consider the set $\mathbb{R}^2$, which consists of 2-tuples of real numbers, equipped with its standard topology. Since time can neither be zero nor negative, we will consider the subset $\mathbb{R}^{2+}\backslash\{0\}$, which consists of all positive real pairs $(\tau_{it-l}, \tau_{it-l-1})$ in $\mathbb{R}^2$ for $l = 0, 1, ..., k$, and equip it with the subspace topology coming from the standard topology on $\mathbb{R}^2$.

It is easy to see that the relation $\sim$ on $\mathbb{R}^{2+}\backslash\{0\}$, defined by: For $x, y \in \mathbb{R}^{2+}\backslash\{0\}$,

$$x \sim y \text{ if } y = \phi x, \text{ where } \phi > 0,$$

is an equivalence relation.[5] Now, consider the function

$$f_i : \mathbb{R}^{2+}\backslash\{0\} \longrightarrow \mathbb{R}^+, \; f_i\left(\tau_{it-l}, \tau_{it-l-1}\right) = \left(\frac{\tau_{it-l}}{\tau_{it-l-1}}\right)^k.$$

[5]Note here that x and y are both pairs of digital times in $\mathbb{R}^{2+}\backslash\{0\}$, e.g., $x = (\tau_{it-l}, \tau_{it-l-1})$ and $y = (\tau_{it-m}, \tau_{it-m-1})$, where l and m are positive integers.

This function is a continuous and invariant under $\sim$. Then, the function

$$\bar{f}_i : \mathbb{R}^{2+}\backslash\{0\}/\sim \quad \longrightarrow \mathbb{R}^+,\ \bar{f}\left([(\tau_{it-l}, \tau_{it-l-1})]\right) = f\left(\tau_{it-l}, \tau_{it-l-1}\right),$$

where $[(\tau_{it-l}, \tau_{it-l-1})]$ is an equivalence class, is a continuous function from the information set $\Omega := \mathbb{R}^{2+}\backslash\{0\}/\sim$ with its quotient topology to $\mathbb{R}^+$ with the standard topology on it. Noting that $\left(\frac{\tau_{it-l}}{\tau_{it-l-1}}\right)^k = \phi_i^k$, the previous analysis demonstrates that the function

$$\bar{f}_i : \Omega \longrightarrow \mathbb{R}^+,\ \bar{f}_i\left([(\tau_{it-l}, \tau_{it-l-1})]\right) = \left(\frac{\tau_{it-l}}{\tau_{it-l-1}}\right)^k$$

is a well-defined function that maps classes of digital times from Ω to $\mathbb{R}^+$. Note that the behavior of $\bar{f}_i$ is governed by the value taken by the autocorrelation coefficient ϕ_i and the distance in time k.

If we further assume that the time series τ_{it-l} is covariance stationary, then the restriction

$$g_i : \Omega \longrightarrow (0, 1],\ g_i\left([(\tau_{it-l}, \tau_{it-l-1})]\right) = \left(\frac{\tau_{it-l}}{\tau_{it-l-1}}\right)^k$$

is a continuous function from the information set Ω to the set of portfolio weights $\mathbb{W}$. Let $l = 0$ for simplicity. Notice that because of the presumption that the digital time process is stationary, i.e., because $0 < \phi_i < 1$, then as $k \longrightarrow 0$, i.e., when the distance between the first and the last visit to the news site is short, which indicates that the investor is actively following asset i's news, then $g_i \longrightarrow 1$, which implies that the investor assigns a high positive weight to asset i in the portfolio. On the other hand, when k is large, i.e., when $k \longrightarrow \infty$, which is indicative of loss of interest in following asset i's news, then $g_i \longrightarrow 0$, which means that the investor is likely to assign a lower weight to asset i or even drop it from his portfolio.[6]

[6]It is worth mentioning that ϕ^k is the autocorrelation function of the AR(1) process,

Thus, for $i = 1, ..., n$, we obtain the following n continuous maps from Ω to $\mathbb{W}$:

$$\begin{aligned}
g_1 &: \Omega \longrightarrow (0,1],\ g_1\left([(\tau_{1t-l}, \tau_{1t-l-1})]\right) = \left(\frac{\tau_{1t-l}}{\tau_{1t-l-1}}\right)^k = \phi_1^k, \\
g_2 &: \Omega \longrightarrow (0,1],\ g_2\left([(\tau_{2t-l}, \tau_{2t-l-1})]\right) = \left(\frac{\tau_{2t-l}}{\tau_{2t-l-1}}\right)^k = \phi_2^k, \\
&\vdots \\
g_n &: \Omega \longrightarrow (0,1],\ g_n\left([(\tau_{nt-l}, \tau_{nt-l-1})]\right) = \left(\frac{\tau_{nt-l}}{\tau_{nt-l-1}}\right)^k = \phi_n^k.
\end{aligned}$$

To complete the proof, we can now define the function $g : \Omega \longrightarrow (0,1]$ as

$$g\left([(\tau_{1t-l}, \tau_{1t-l-1})], [(\tau_{2t-l}, \tau_{2t-l-1})], ..., [(\tau_{nt-l}, \tau_{nt-l-1})]\right) = (g_1, g_2, ..., g_n).$$

This function is a continuous function that maps pairs of classes digital times, as proxies of information updates, to the set of portfolio weights $\mathbb{W}$. ■

3 Empirical Analysis and Evidence

The previous proposition shows the existence of a continuous mapping from the investor's information set Ω to the set of portfolio weights $\mathbb{W}$. Updates in Ω changes the portfolio weights in $\mathbb{W}$. This, in turn, alters the demand for the corresponding financial assets causing their market value to be above or below the book value. Therefore, if we take p to be a market portfolio, which consists of n assets, the previous proposition suggests that the market rate of return, $r_p := w_1 r_1 + w_2 r_2 + ... + w_n r_n$, is influenced by changes in its market-to-book value, $\triangle(M/B)$, which, in turn, are induced by changes in Ω. More precisely, the market rate of return continuously transitions from a lower inefficient regime where asset prices are undervalued to an upper

which in this case, can be shown to be bounded above by 1 and below by 0.

inefficient regime where asset prices are overvalued, and in between these two inefficient regimes the market rate of return passes briefly by an episode of efficiency, where its market value is equal to its book value.

The previous dynamic of the market rate of return r_p is best captured by a regime switching model with $\triangle(M/B)$ as transition or threshold variable. In particular, we will employ the logistic smooth transition regression model of order 1 (LSTR(1)) pioneered by Granger and Teräsvirta (1993) and Teräsvirta (1994).[7] The smooth transition regression (STR) model is best suited here because it is capable of describing processes that can move from one regime to another such that the transition is *smooth*. By applying such a model to the rate of return of a capital market portfolio, one can capture the switching behavior of the market rate of return between episodes of efficiencies and inefficiencies. Moreover, the degree of smoothness of this switching behavior can be taken to reflect the speed of the adaptability process, i.e., how fast investors react to financial news; a smooth transition reflects a slower steady adaptability whereas an abrupt transition from one regime to another is indicative of a faster and more swift adaptability to financial news. The selection of the transition variable in the STR model is crucial as it explains the dynamics of the market rate of return. The convention in the regime switching literature is to use a lag order autoregressive component of the dependent variable, i.e., r_{t-d}, where d is a delay parameter, as transition variable. In contrast to those studies, this paper shows that the best transition variable that is capable of explaining the dynamic of the market rate of return is an exogenous stationary transition variable, namely, the first difference of the one-period lag of the market-to-book ratio, denoted by $\triangle(M/B)_{t-1}$ in the text. Taking the first difference of the one-period lag of M/B, rather than just the first difference of the variable, is sensible as it is consistent with the presumption that updates in Ω in period $t-1$, as

[7] STR models have been used extensively in the regime switching literature. For a recent review, see Fahmy (2014) and the references therein.

reflected in the change of the M/B in period $t-1$, have an impact on the market rate of return in the following period, i.e., in period t.

Before we show the results, a few words regarding the STR model employed in this paper are in order. In a compact notation, the standard STR model of order p is expressed as follows.

$$r_t = \boldsymbol{\beta}^{'}\mathbf{r}_t + \boldsymbol{\Theta}^{'}\mathbf{r}_t G\left(s_t;\gamma,\mathbf{c}\right) + \varepsilon_t, \qquad t = 1,2,...,T, \tag{3}$$

where r_t is the market rate of return at time t, $\mathbf{r}_t^{'} = (1, r_{t-1}, ..., r_{t-p})$, $\boldsymbol{\beta}^{'} = (\beta_0, \beta_1, ..., \beta_p)$, and $\boldsymbol{\Theta}^{'} = (\theta_0, \theta_1, ..., \theta_p)$ are parameter vectors, $\varepsilon_t \sim i.i.d.(0, \sigma^2)$, and $G(s_t;\gamma,\mathbf{c})$ is a transition function. It is a bounded function (between 0 and 1) of the continuous transition variable s_t, which will be taken to be $\triangle\left(M/B\right)_{t-1}$, and continuous everywhere in the parameter space for any value of s_t. The transition function $G(\cdot)$ is a *logistic function* defined in general as

$$G(s_t;\gamma,\mathbf{c}) = \left(1 + \exp\{-\gamma \prod_{i=1}^{k}(s_t - c_i)\}\right)^{-1}, \qquad \gamma > 0, \tag{4}$$

where γ is the slope of the function, and $\mathbf{c} = (c_1, ..., c_k)^{'}$ is a vector of location parameters such that $c_1 \leq \cdots \leq c_k$. Given such definition, the STR model defined in (3) is then referred to as the logistic smooth transition regression (LSTR) model.

The transition function in (4) is a monotonically increasing function of the transition variable s_t. The restrictions $\gamma > 0$ and $c_1 \leq \cdots \leq c_k$ are identifying restrictions. The choice of k determines the behavior of the logistic transition function. Two common choices for k are used in the literature: $k = 1$ and $k = 2$. In the LSTR model with $k = 1$ (LSTR(1)), the parameters vectors change monotonically as a function of s_t from $\boldsymbol{\beta}$ to $\boldsymbol{\beta} + \boldsymbol{\Theta}$. This reflects the capability of the LSTR(1) model to characterize processes whose dynamic properties are different in an upper regime from what they are in a lower regime such that the transition between the two regimes is smooth. The

LSTR model with $k = 2$ give rise to three regimes; two outer regimes and a middle one. Our results show, as we will see shortly, that the logistic function in the fitted STR model to the market rate of return is of order 1. This is consistent with the earlier proposition, which suggests that the market rate of return displays *two* distinct inefficient regimes. The logistic function of order 1 takes the form

$$G(s_t; \gamma, c) = \left(1 + \exp\{-\gamma(s_t - c)\}\right)^{-1}, \qquad \gamma > 0. \tag{5}$$

Note that when $s_t \longrightarrow -\infty$, $G(\cdot) = 0$; this defines the lower regime. On the

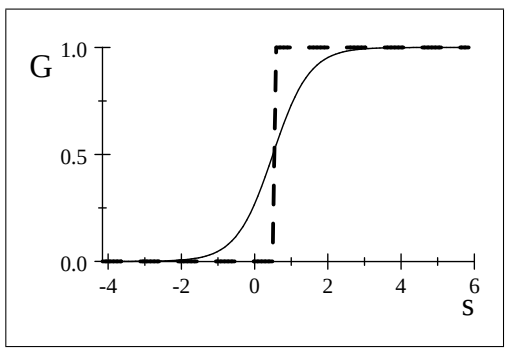

Figure 1: The smooth transition logistic function of order 1 with a moderate slope $\gamma = 2$ (the solid line) and with an extremely larger slope $\gamma = 1000$ (the dashed line). The threshold value $c = 0.5$.

other hand, when $s_t \longrightarrow +\infty$, $G(\cdot) = 1$ and the time series is said to be in an upper regime. The first-order logistic function in (5) is depicted in Figure 1, where the threshold $c = 0.5$ and the slope $\gamma = \{2, 1000\}$ for the solid and the dashed lines respectively.

3.1 Data Description

The market rate of return studied in this paper is the Standard and Poor (S&P500) index over the period between the fourth quarter of 1969 and the second quarter of 2014. A plot of the index rate of return, denoted by r_t in

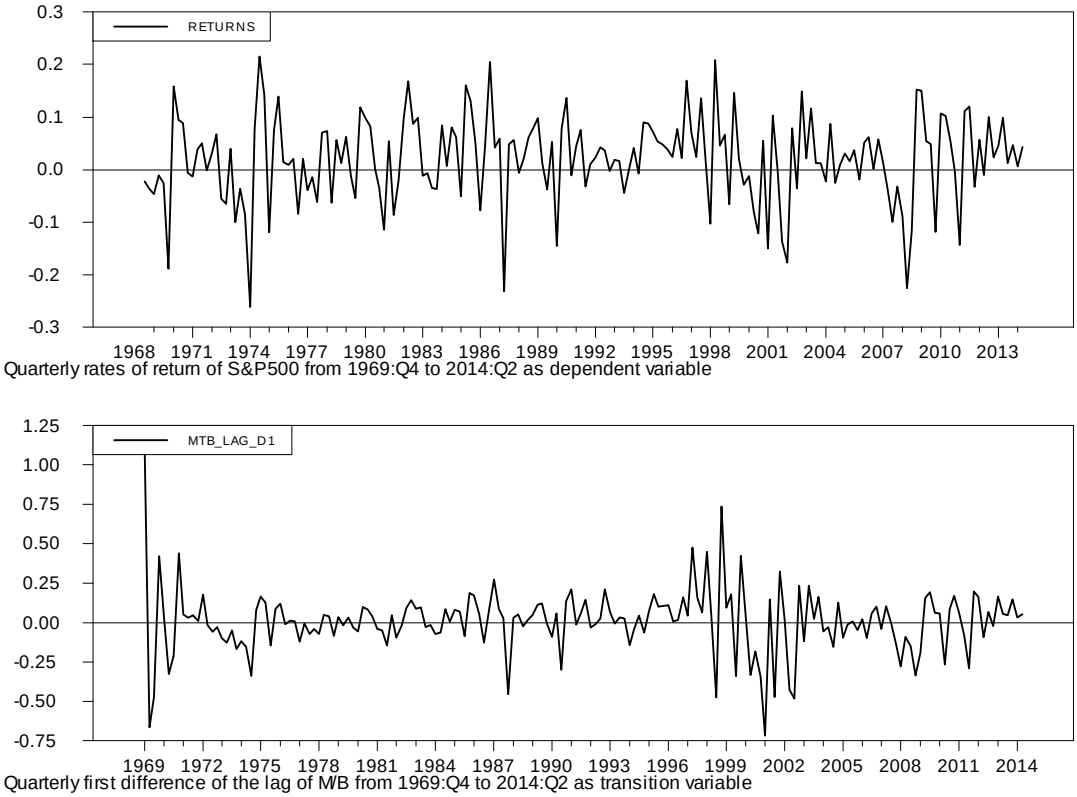

Figure 2: The quarterly rates of return on the S&P500 and the first difference of the one-period lag of the market-to-book ratio between 1969:Q4 and 2014:Q2.

the text and defined as $r_t = \frac{P_{t+1}-P_t}{P_t}$, where P_t is the end-of-quarter closing price of the S&P500 composite index taken from Compustat and Center of Research on Security Prices (CRSP) merged file data base, is depicted in the top panel of Figure 2. The transition variable is the first difference of the one-period lag of the market-to-book ratio, denoted by $\triangle (M/B)_{t-1}$ and defined as $\triangle (M/B)_{t-1} = \left(\frac{M}{B}\right)_{t-1} - \left(\frac{M}{B}\right)_{t-2}$.

The quarterly data on closing prices, shares outstanding, and shareholder's equity of all underlying constituents of the S&P500 since 1969 was used to construct the M/B ratio using the following guidelines. First, any asset that was not included in the S&P500 in period t, i.e., quarter t, was dropped from the data set. Second, any asset that showed no value for either its stock price or the shareholder's equity, or the number of shares outstanding was also dropped from the computations. Finally, the market value at period t, M_t, was computed as the sum of all shares outstanding for asset i times the quarter closing price of asset i, for all assets included in the index in period t. As for the book value at period t, B_t, it is simply the sum of shareholder's equity for all assets included in the index in period t. The transition variable $\triangle (M/B)_{t-1}$ is depicted in the bottom panel of Figure 2.

In order for the series under consideration to be able to fluctuate between the two upper and lower (inefficient) regimes, they need to have some sort of mean reversion mechanism, and hence, they ought to be stationary. The stationarity of r_t and $\triangle (M/B)_t$ is confirmed at the 5% level of significance by the ADF, PP, and KPSS tests as shown from Table 1. This is consistent with our hypothesis that the market rate of return is pushed by changes in its market-to-book value from one inefficient regime to another passing by a brief episode of efficiency.

A quick look at Figure 1, one can notice that the transition variable is capable of capturing every bear and bull episode of the S&P500 over the given period. For instance, the crash of 1969-1970 that resulted from the drop in consumer confidence as a result of the high inflation and the increased

deficits from the Vietnam War is reflected in the two swings in the transition variable, i.e., $\triangle (M/B)_{t-1}$, during the few quarters between 1969 and 1970. The financial crash of 1974 that followed the OPEC oil embargo in 1973 is well noticed in the downfall of the S&P500 returns as shown in the upper panel of Figure 2. This swing is also captured by the transition variable in the lower panel. The crash of 1987, which followed the contractionary monetary policy adopted by the Federal Reserve in 1986 to combat inflation, is also picked up by the transition variable. The so-called "Dot-Com bubble" effect that caused the clear downfall of the S&P500 in the second half of 2002 as shown in the upper panel of Figure 2 was also captured by the M/B ratio as shown from the down spike in 2001. The recent Housing Bubble that bursted in 2008 is also clearly captured by the transition variable. The variable seems also to capture the bull episodes in the S&P500 quite well, and thus, acts as a good transition variable in the STR model.

3.2 The LSTR(1) Model Applied to the S&P500 Returns

Following the modelling framework proposed by Granger and Teräsvirta (1993), Teräsvirta (1994), and Eitrheim and Teräsvirta (1996), which consists of three stages, we begin by specifying an adequate linear $AR(m)$ model for r_t, where m is the value that minimizes the Akaike (1974) information criterion (AIC). Before accepting the suggested $AR(m)$ model as the starting point of the regime switching analysis, preliminary diagnostic tests should be applied to the preliminary model in order to ensure its adequacy as a starting linear model. In particular, the Ljung-Box (1978) test of no serial correlation of order $q = 1$ up to $q = 4$ in the residuals and Engle's (1982) Lagrange multiplier (LM) test of no autoregressive conditional heteroskedasticity (ARCH) of order $v = 1$ up to $v = 4$ in the residuals are considered.

The AIC for r_t was minimized at a lag order $m = 1$, and therefore, the preliminary linear model for the nonlinearity analysis was an AR(1) model.

The results of the estimated preliminary AR(1) model and the previous misspecification tests are summarized as follows:

$$
\begin{aligned}
r_t &= \underset{(0.007)}{0.02} + \underset{(0.07)}{0.087} r_{t-1} + \widehat{a}_t, \\
Q(1) &= 0.96, \qquad Q(8) = 0.76, \\
LM_{ARCH(1)} &= 0.21, \qquad LM_{ARCH(4)} = 0.09, \\
JB &= 0.08, \qquad K_3 = -0.46,\ K_4 = 3.64,
\end{aligned}
$$

where $\widehat{a}_t$ is the series of residuals, K_3 and K_4 are skewness and kurtosis respectively, JB is the p-value of the Jarque-Bera (1987) test of normality, and the figures in parentheses beneath the estimated parameters are the standard errors. Judging by the p-value of the Ljung-Box (1978) test, the null hypothesis of no serial correlation of order $q = 1$ up to $q = 8$ in the residuals series was not rejected at the 5% level of significance. Also, judging by the p-value of Engle's (1982) test, the null hypothesis of no ARCH of order $v = 1$ up to $v = 4$ was not rejected at the 5% level of significance. Finally, the null hypothesis of normality of errors was not rejected at the 5% level of significance as seen from the p-value of the JB test. Judging by the previous tests, the AR(1) model passes all preliminary diagnostic tests and can act as a good starting model for the nonlinearity analysis.

The next step in the specification stage is *nonlinearity testing*. The null hypothesis of linearity, denoted by H_{0L} in the text, is tested against the alternative of a nonlinear STR model. We shall adopt the nonlinearity test suggested by Teräsvirta (1994) and Luukkonen et al (1988), which is basically an LM test with an asymptotic F distribution when H_{0L} is valid. The authors also suggested a sequence of F tests, denoted by F_2, F_3, and F_4 in the text, to choose an appropriate type of STR model, i.e., LSTR(1) or LSTR(2). In summary, the purpose of nonlinearity testing is twofold: First, it allows the researcher to identify which transition variable, in a set

of transition candidates, is suitable for the STR model. Second, the F-tests procedure identifies what type of STR model fits the data best, i.e., LSTR(1) or LSTR(2).[8]

The convention in the regime switching literature is to specify a set of candidate transition variables, e.g., time trend, autoregressive lags of the dependent variable, or other sensible exogenous threshold variables, and repeat the nonlinearity test for each transition candidate in the transition set. If the null hypothesis of linearity, H_{0L}, using the F test, denoted by F_L in the text, is rejected for at least one of the models, the model against which the rejection, measured in the p-value, is strongest is chosen to be the STR model to be estimated. Although, our earlier proposition suggested that $\triangle (M/B)_{t-1}$ is the potential transition variable in the STR model, for the sake of consistency, we performed the nonlinearity test for a time trend, one period lag of r_t, and for $\triangle (M/B)_{t-1}$. The model against which the rejection was strongest was the STR model with $\triangle (M/B)_{t-1}$ as threshold variable and the selected STR model was the LSTR(1) model as shown from the nonlinearity tests results in Table 2.

After determining the transition variable and the type of the STR model, the next step is *estimation*. The parameters of the STR model in (3) are estimated using conditional maximum likelihood. The log-likelihood function of the STR model is

$$l(\boldsymbol{\beta}, \boldsymbol{\Theta}, \sigma, \gamma, \mathbf{c}) = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\sigma^2) - \frac{1}{2}\frac{\{r_t - (\boldsymbol{\beta}' + \boldsymbol{\Theta}' G(s_t; \gamma, \mathbf{c}))\mathbf{r}_t\}^2}{\sigma^2}. \quad (6)$$

Conditional on starting values of the parameters, the log-likelihood function in (6) is maximized using the iterative Broyden-Fletcher-Goldfarb-shanno (BFGS) algorithm. Finding good starting values is important for the algorithm to work properly.

Starting values are obtained by constructing a grid in γ and $\mathbf{c}$, estimating

[8] The nonlinearity test for the STR model and the selection criterion is described in detail in Teräsvirta (1994).

the parameter vectors $\boldsymbol{\beta}$ and $\boldsymbol{\Theta}$ conditionally on (γ, c) for $k = 1$ (LSTR(1) model), and computing the sum of squared residuals. The parameter values that correspond to the minimum of that sum are taken as the starting values.[9]

The last stage in the modelling cycle is the *diagnostic stage*, where the adequacy of the fitted model is considered. The misspecification tests for the STR models that have been considered in Eitrheim and Teräsvirta (1996) and Teräsvirta (1998) will be considered in this paper. In particular, three tests will be considered. The first test is an LM-type test of no error autocorrelation of order q. The p-value of the test is denoted by $LM_{AUTO(q)}$. The second diagnostic test is Engle's (1982) Lagrange multiplier (LM) test of no autoregressive conditional heteroskedasticity (ARCH) of order $v = 1$ up to $v = 4$ in the residuals. The last diagnostic test is a parameter constancy, where the null hypothesis of parameter constancy in the STR model is tested against non-monotonic change, non-monotonic symmetrical change, and non-monotonic and non-symmetrical change with p-values denoted by $PC1$, $PC2$, and $PC3$ respectively.

The fitted LSTR(1) model and the results of Eitrheim and Teräsvirta (1996) and Teräsvirta's (1988) misspecification tests are reported as follows.

$$r_t = \underset{(0.098)}{0.32}\, r_{t-1} + \left\{ \underset{(0.009)}{0.034} - \underset{(0.15)}{0.54} r_{t-1} \right\} \left(1 + \exp\left\{ -\underset{(702)}{241} \left(\frac{\triangle (M/B)_{t-1} - \underset{(0.0034)}{0.03}}{0.18} \right) \right\} \right)^{-1} + \widehat{\varepsilon}_t,$$

$$\widehat{\sigma}_{M/B} = 0.18, \qquad \widehat{\sigma} = 0.08,$$

$$LM_{ARCH(1)} = 0.16, \qquad LM_{ARCH(4)} = 0.06,$$

$$LM_{AUTO(1)} = 0.72, \qquad LM_{AUTO(4)} = 0.94, \qquad LM_{AUTO(8)} = 0.89,$$

$$JB = 0.051, \qquad K_3 = -0.34, \qquad K_4 = 3.58,$$

[9] To facilitate the construction of an effective grid, I follow Teräsvirta's (1998) suggestion to standardize the exponent of the transition function $G(s_t; \gamma, \mathbf{c})$ by dividing it by the kth power of the sample standard deviation of the transition variable $\widehat{\sigma}_s^k$. This is done mainly to render the parameter γ scale-free.

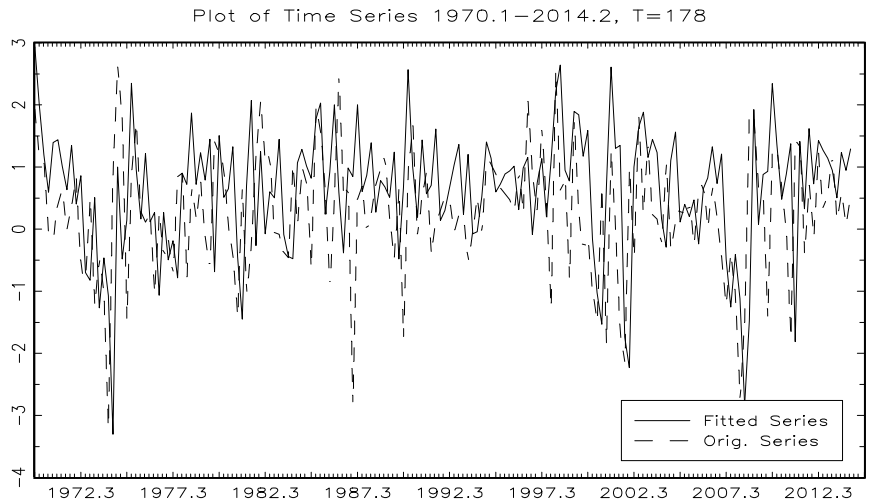

Figure 3: Original and fitted values of r_t between 1969:Q4 and 2014:Q2 with the first difference of the one-period lag of the market-to-book ratio as transition variable.

$$PC(1) = 0.10, \qquad PC(2) = 0.05, \qquad PC(3) = 0.06, \tag{7}$$

where $\widehat{\sigma}_{M/B}$ is the sample standard deviation of the transition variable $s_t = \triangle (M/B)_{t-1}$ and $\widehat{\sigma}$ is the residual standard deviation. The figures in parentheses beneath the parameter values are standard errors. $LM_{AUTO(v)}$ is the p-value for the vth order autocorrelation. $LM_{ARCH(q)}$ is the p-value of the qth order ARCH. $PC1, PC2$, and $PC3$ are p-values for parameter constancy tests against monotonic change, non-monotonic symmetrical change, and non-monotonic and non-symmetrical change respectively. The original and fitted series from the STR model of r_t are plotted in Figure 3.

Perhaps the most noticeable detail of equation (7) is the large standard deviation of the estimated slope of the logistic transition function $\widehat{\gamma} = 241$. It is common for LSTR models that the estimated standard deviation of γ tends to be large for large values of γ. This is not crucial, however, as it does not affect either the shape of the logistic function $G(\triangle(M/B)_{t-1}, \gamma, c)$ or the other estimates of the model. The large slope of the transition function, as

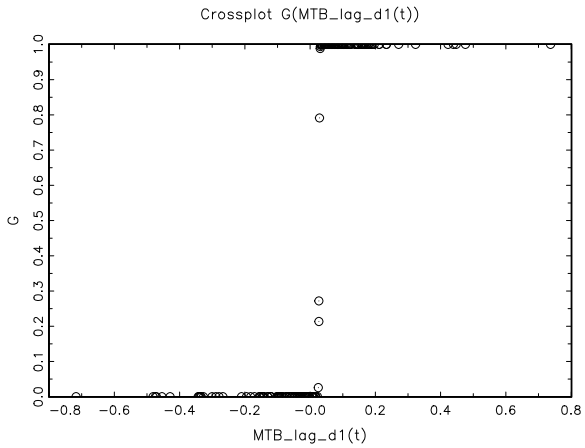

Figure 4: Transition function $G(\triangle(M/B)_{t-1}, \gamma, c)$ as a function of observations. Each dot corresponds to an observation. The transition variable is the first difference of the one-period lag market-to-book ratio.

depicted from the dot plot in Figure 4, where each dot corresponds to one observation, is indicative of a fast transition of r_t from one inefficient regime to another. This is consistent with the AMH, which posits that market participants adapt to changing market conditions. The results also hint to a swift adaptability process; that is, the S&P500 returns series continuously fluctuates from an upper inefficient regime to a lower inefficient regime passing quickly by an efficient regime. Hence, the market is efficient sometimes but inefficient most of the times.

Other than the large slope of the transition function, the estimated coefficients are all significant and the model passes the misspecification tests. One exception is the almost rejection of the null-hypothesis of normality of errors at the 5% level of significance as seen from the p-value of the JB test statistic. But, this is due to the large outlier of the 1987 crash, where the absolute of the standardized residuals is greater than three as seen from the plot of the standardized residuals in Figure 5.

The dynamic behavior of the market rate of return can be detected by

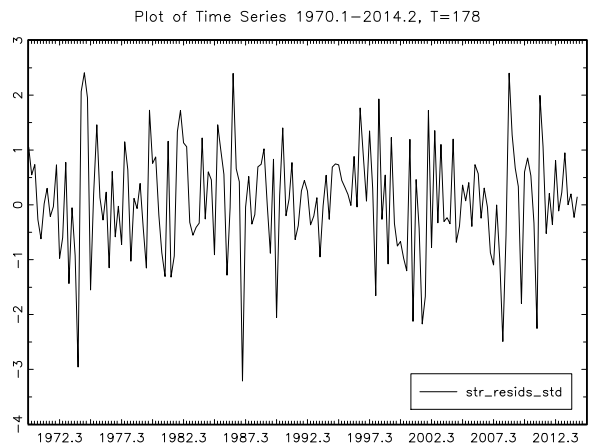

Figure 5: The standardized residuals of the LSTR(1) model in the period between 1969 and 2014 with the first difference of the one-period lag market-to-book ratio.

examining the transition function and the behavior of the transition variable above and below the estimated threshold value $\widehat{c} = 0.03$, which is approximately equal to zero. This means that if the transition variable $\triangle (M/B)_{t-1} > 0.03$, or equivalently, if approximately $\left(\frac{M}{B}\right)_{t-1} > \left(\frac{M}{B}\right)_{t-2}$; that is, if the asset prices are *overvalued*, then the transition function $G(\cdot)$ will approach 1 and the market rate of return will move to an upper stationary inefficient AR(1) regime such that

$$r_t = 0.034 - 0.22 r_{t-1} + \widehat{\varepsilon}_t$$

as demonstrated in Figure 6 where the estimated threshold value of 0.03 is the doted line in the upper panel of the figure. On the other hand, if the transition variable is below 0.03, or approximately, if $\left(\frac{M}{B}\right)_{t-1} < \left(\frac{M}{B}\right)_{t-2}$; that is, if asset prices are undervalued, then the transition function $G(\cdot)$ will approach 0 and the market rate of return will move to a lower stationary

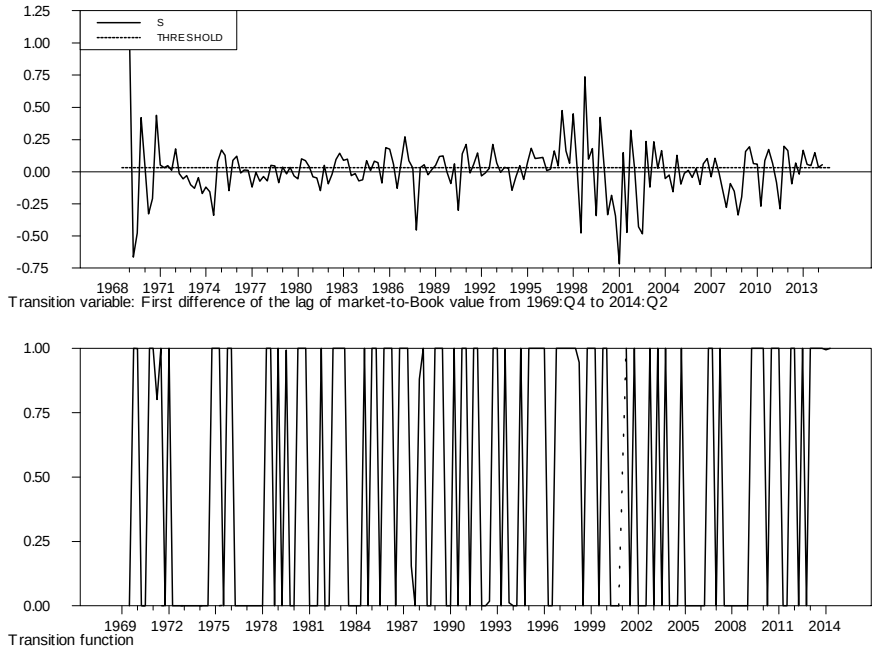

Figure 6: A plot of the transition variable $\triangle (M/B)_{t-1}$ and the transition function $G_t \left(\triangle (M/B)_{t-1}; \gamma, c\right)$ between 1969 and 2014.

inefficient AR(1) regime such that

$$r_t = 0.32 r_{t-1} + \widehat{\varepsilon}_t.$$

The behavior of the S&P500 rate of return in both inefficient regimes is summarized in Table 3. A simulation of both regimes is also given in Figure 7. Notice that the market rate of return times series wanders in both regimes but always revert back to the mean because the series is stationary. Notice also that the efficient regime is embedded in between the upper and lower inefficient regimes because of the swift swings from one inefficient regime to the other.

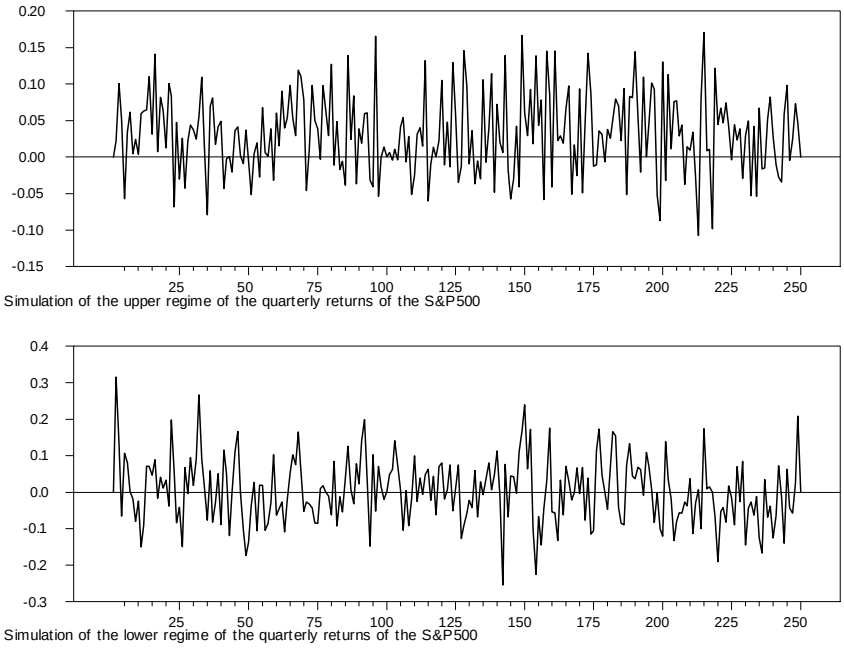

Figure 7: Simulation of the upper AR(1) stationary regime of the quarterly S&P returns, $r_t = 0.034 - 0.22 r_{t-1} + \widehat{\varepsilon}_t$, and the lower AR(1) stationary regime, $r_t = 0.32 r_{t-1} + \widehat{\varepsilon}_t$, based on 250 random realizations.

4 Concluding Remarks

In this paper, we prove mathematically the claim that the observed irrationality in investors' behavior in making financial decisions concerning allocation of assets and construction of portfolios, which is due to continuous updates in their information sets, causes investors to continuously alter the composition of their portfolios, which, in turn, causes the market value of financial assets to be above or below its book value. In particular, we show the existence of a continuous map from the investors' information set to the market-to-book values of the individual assets traded in capital markets. The rationale is that continuous updates of the information set induces episodes of inefficiencies, where asset prices are undervalued or overvalued. Since any financial market is seen as a portfolio of assets and since the market rate of return is the weighted sum of these individual assets, then the previous proposition means that the market rate of return continuously switches from a lower inefficient regime where asset prices are undervalued to an upper inefficient regime where asset prices are overvalued, and in between these two inefficient regimes the market rate of return passes briefly by an episode of efficiency, where its market value is equal to its book value. Hence, financial markets are efficient sometimes and inefficient most of the time.

In addition to proving the previous proposition mathematically, the paper also confirms its empirical validity by following a regime switching approach in capturing the dynamics of the S&P quarterly rates of return over the period between 1969 and 2014. In summary, the empirical analysis confirms four main implications of the previous proposition: (1) The adaptability process as suggested in the previous proposition implies that the two inefficient regimes are both stationary. Otherwise, the market rate of return can wander in one regime without ever returning back to the other. Our results confirm that; both upper and lower (inefficient) regimes were stationary. (2) The adaptability process entails a switching behavior between inefficient regimes. This implies that the market rate of return time series is non-linear

and is best modelled by a regime switching model. Our nonlinearity test and the regime switching framework confirm that. (3) The transition of the market rate of return from one inefficient regime to another over time is the result of an observed exogenous factor that is capable of capturing the irrational behavior of market participants, namely, the first difference of the one-period lag market-to-book ratio. The nonlinearity tests sequence confirms that. (4) Finally, the transition between the two inefficient regimes could be smooth or swift depending on the adaptability process of investors. This is consistent with Lo's AMH, which posits that financial markets witness episodes of inefficiencies, but the adaptability of its participants forces them to revert back to efficiency. Our results also confirm that.

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Table 1: The stationarity of the Standard and Poor returns and the first difference of the market-to-book ratio over the period between 1969 and 2014.

Time Series	ADF(k)	PP	KPSS
r_t	$-10.56(1)$	-15.51	0.08
$\triangle (M/B)_t$	$-9.66(1)$	-12.42	0.09

ADF, PP, and KPSS are respectively the augmented Dickey-Fuller (1979), the Phillips-Perron (1988), and the Kwiatkowski, Phillips, Schmidt, and Shin (1992) tests with a trend. k is the number of lags of the ADF test. The 5% critical values are -3.45 for ADF and PP and 0.146 for KPSS.

Table 2: P-values of the linearity F-tests sequence applied to the rates of return of the Standard and Poor with the first difference of the one-period lag market-to-book ratio as exogenous transition variable.

s_t	F_L	F_4	F_3	F_2	Suggested Model
r_{t-1}	9.51×10^{-2}	4.24×10^{-1}	5.71×10^{-1}	1.97×10^{-2}	Linear
$TREND$	9.78×10^{-1}	6.92×10^{-1}	8.27×10^{-1}	9.76×10^{-1}	Linear
$\triangle (M/B)_{t-1}$	9.67×10^{-3}	1.81×10^{-2}	3.64×10^{-2}	3.37×10^{-1}	LSTR(1)

Table 3: The upper and lower regimes of the LSTR(1) model fitted to the rates of return of the Standard and Poor with the first difference of the one-period lag market-to-book ratio as exogenous transition variable.

$s_t = \triangle (M/B)_{t-1}$	Upper regime: $G(\cdot) = 1$	Lower regime: $G(\cdot) = 0$
Threshold: $\widehat{c}$	0.03	0.07
Model	$r_t = 0.034 - 0.22 r_{t-1} + \widehat{\varepsilon}_t$	$r_t = 0.32 r_{t-1} + \widehat{\varepsilon}_t$
Behavior of r_t	$Stationary\ AR(1)$	$Stationary\ AR(1)$
Mean	$E(r_t) = 0.0278$	$E(y_t) = 0$
Variance	$var(r_t) = 0.0036$	$var(y_t) = 0.007$