Predicting Sovereign Fiscal Crises: High-Debt Developed Countries

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Abstract

Every country has a fiscal limit on debt, where that limit represents a debt level so high that the country’s economic and political systems cannot raise taxes or reduce spending sufficiently to maintain solvency. We estimate fiscal feedback rules for ten high-debt, developed countries in an attempt to identify fiscal limits. Our estimates imply that a fiscal feedback rule cannot reveal a fiscal limit for a country that has not experienced a crisis. However, estimates of long-run debt, together with debt history, can be combined to yield an estimate of a lower bound on the fiscal limit. We use estimates of stable fiscal feedback rules for ten high-debt developed countries to project debt forward from 2008, and compare the projections with our estimates of the lower bound on debt. We label countries, whose debt projections exceed the lower bound as high-risk. Both Greece and Portugal enter the high-risk category about two years prior to their fiscal crises. At least four of the remaining eight countries are at high risk in 2014, warning of potential future crises. Canada and Belgium retain safe status due to their ability to quickly check explosive debt behavior following a large surplus shock.

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1 Introduction

Greece lost access to credit markets when debt exceeded 130% of GDP, whereas Belgium successfully handled identical debt relative to GDP. What determines the value of debt beyond which a country loses access to financial markets? Can we estimate this upper bound on debt for a country which has never lost access? We use historical data on primary surpluses and debt relative to GDP for ten high-debt, developed countries, two of which lost access to markets after the 2008 global financial crisis, to estimate the behavior of the primary surplus to debt. The high-debt countries include Belgium, Canada, France, Greece, Italy, Japan, Portugal, Spain, the US, and the UK. We use our estimates to determine to what extent the data can provide values for the upper bound on debt and to make assessments of the risk of fiscal insolvency.

The literature offers two concepts of an upper bound on debt. Davig, Leeper and Walker (2011), and Bi (2012) argue that there is an upper bound on the level of debt that a country can service and/or repay, and they define this as the "fiscal limit" on debt, typically expressed as a fraction of GDP. They motivate the limit by the top of the Laffer curve for distortionary taxes. However, the concept can be more general. A fiscal limit on debt can include the inability to reduce government spending, perhaps due to the dependence of economic activity on the provision of public goods, together with the inability to raise tax rates for other reasons, including tax evasion. If the attempt to raise the primary surplus sufficiently to service debt causes output to fall proportionately more than debt falls, then the country has hit its fiscal limit on debt. Bi, Leeper, and Leith (2013) argue that the fiscal limit could also be political, whereby the democratic process is not able to raise the primary surplus sufficiently to service the debt.

The second concept is based on the stability properties of the set of equations, governing the country’s evolution of debt, and its fiscal feedback rule for adjusting the primary surplus to lagged debt. Ghosh et al. (2013) argue that once the value of debt enters an unstable region in which debt is explosive upwards, debt is expected to violate any fiscal limit. The market would not lend into this region, establishing the value of debt on the boundary of this unstable region as a "debt limit". This debt limit is therefore a property of the fiscal feedback rule, implying that we can estimate its value by estimating the fiscal feedback rule. Ghosh et al. (2013) identify the debt limit using a regression of the primary surplus on powers of debt, up to the cubic power, in a panel model. A negative coefficient on the cubic power allows them to identify the value of debt beyond which debt explodes, the debt limit. They label the reduced responsiveness of the primary surplus to debt as
debt becomes high "fiscal fatigue."\textsuperscript{1}

Our first attempt to estimate upper bound on debts uses the Ghosh et al (2013) concept of the debt limit as the value of debt at which the dynamic system in debt and the surplus becomes unstable. We estimate individual-country fiscal feedback rules based on our hypothesis that countries could have different ability to handle debt due to different fiscal policy. We estimate a threshold model, allowing the surplus-responsiveness to debt to change as debt crosses thresholds. We find behavior consistent with fiscal fatigue for only three out of the ten countries, including Greece, the US, and the UK. We do not find fiscal fatigue for the remaining seven countries, implying that the regressions cannot identify debt limits for these countries. These results confirm our hypothesis that behavior is highly diverse across countries, with some countries exhibiting fiscal fatigue while most do not. However, even for countries for which we can identify debt limits, our estimated debt limits are too low to be compatible with lending behavior over time. Countries were able to participate in financial markets when their debt was above the limits we estimate.

The ability to identify a debt limit from the responsiveness of the primary surplus to debt relies on the assumption that responsiveness depends on the level of debt. However, it is possible that the time variation in the primary-surplus responsiveness to debt is not due to the level of debt itself, but to something else which is correlated with debt. If responsiveness is increasing in the interest rate and not in debt, then the reduced responsiveness at high debt, that we find for Greece, the US, and the UK, and that Ghosh et al. (2013) find in their panel, could occur because debt reached high values when interest rates were low. Our estimates confirm that all countries raise (lower) responsiveness with an increase (decrease) in the interest rate. Responsiveness is highest in the period when interest rates are highest. Therefore, the model with responsiveness varying with the interest rate tells a more consistent story across countries than the model with responsiveness varying with the level of debt. However, if responsiveness varies with the interest rate and not with debt, then we cannot obtain a value of debt at which the stability properties change. The interest-rate model does not provide an estimate of the debt limit.

After failing to find reliable estimates of debt limits, we turn to the alternative concept of an upper bound on debt, the fiscal limit, defined as the largest value of debt the country can service and/or repay. We use our interest-rate model together with additional information to estimate a lower bound on the fiscal limit. First, our estimates for the long-

\textsuperscript{1}"Fiscal fatigue" is an empirical concept without explicit economic justification. The same economic fundamentals could be responsible for both the upper bound on the ability to repay and fiscal fatigue.
run value of debt must be below the fiscal limit since agents were lending in periods when
debt was moving toward these estimated long-run values. Second, any debt level, which a
country successfully services while remaining in the market, must also be below the fiscal
limit. Since there were no fiscal crises in this set of countries between 1970 and 2007, then
each country’s fiscal limit must be above the maximum value of debt observed over that
period. We use the largest of these two debt measures, the estimated long-run value of
debt and the historical maximum, to construct a lower bound on the fiscal limit.

We project debt forward from 2008, using our estimates of stable fiscal feedback rules,
and separate countries into high and low risk categories depending on whether the peak of
the debt projection is greater than or less than the lower bound. We begin the assessment
in 2008, and update the lower bounds over time, as countries successfully service increasing
levels of debt, placing countries into high and low risk categories in subsequent years.
We find that Greece and Portugal entered the "high risk" category in 2008 and 2009
respectively. Both countries lost access to financial markets two years later. Actual fiscal
limits on debt were between 111.5% and 126.78% of GDP for Greece, and near 97.68% of
GDP for Portugal. We find that Spain entered the high risk category in 2012. Although
Spain did not actually lose access to the markets, it experienced very high market interest
rates and received an official European loan in 2012. We find that France, Italy, Japan,
the UK, and the US all joined the high-risk category, with the US exiting the high-risk
category in 2013. Countries, which remain at potential risk of a fiscal crisis in 2014,
include France, Italy, Japan, Spain, and possibly the UK.

In contrast, Belgium and Canada are at low risk even though neither has particularly
low debt relative to others. These two countries differ from the others in their ability to
return quickly to their estimated fiscal feedback rules following a large financial shock,
thereby setting debt relative to GDP on a stable path to its long-run equilibrium value.
The ability to immediately check explosive behavior of debt, following a large financial
shock, sets these countries apart.

This paper is organized as follows. The next section derives criteria for solvency
implied by fiscal limits. Section 3 is empirical, with the first part providing estimates
with the responsiveness varying with the level of debt, and the second estimates with
the responsiveness varying with the interest rate. Section 4 uses our estimates of the
interest-rate model to assess risk following the global financial crisis for our ten high-debt
developed countries. Section 5 provides conclusions.
2 Fiscal Limits and Solvency

We assume that each country faces a fiscal limit on debt, defined as the largest value of debt that a country could service and/or repay. The concept is about ability-to-pay, not willingness. We are therefore assuming that the government can commit to pay that which it is able. We show below that a government is both solvent and able to borrow when debt remains below an "effective fiscal limit," which we derive.

2.1 Effective Fiscal Limit

2.1.1 Fiscal Policy and Government Debt

We allow the fiscal rule governing the evolution of the primary surplus to depend on its lagged value, consistent with empirical evidence.\(^2\) We assume that fiscal policy is characterized by a simple fiscal feedback rule whereby the primary surplus as a fraction of GDP \((s_t)\) reacts to its own lag \((s_{t-1})\) and to lagged debt as a fraction of GDP \((d_{t-1})\), given by

\[
s_t = c + \beta s_{t-1} + \gamma d_{t-1} + \epsilon_t, \tag{1}
\]

where \(c\) is a constant governing the long-run value of the primary surplus relative to GDP, and \(\epsilon_t\) represents shocks bounded on the interval \((-\bar{\epsilon}, \bar{\epsilon})\), possibly from the business cycle. The assumption of boundedness is reasonable given that surplus shocks are measured as a fraction of GDP. In the remainder of the paper we refer to the primary surplus as a fraction of GDP and debt as a fraction of GDP, more simply, as the surplus and debt, respectively.

Debt accumulates according to

\[
d_t = \delta_t (1 + r_{t-1}) d_{t-1} - s_t,
\]

where \(\delta_t\) denote the fraction of debt that is repaid and \(r_{t-1}\) the domestic growth-adjusted interest rate set in period \(t - 1\) for payment in period \(t\).

We assume that the government borrows in a global capital market with equilibrium characterized by interest rate parity, requiring

\[
1 + r = (1 + r_{t-1}) E_{t-1} \delta_t,
\]

where \(r\) denotes the world growth-adjusted risk-free rate, which we refer to in the remain-

\(^2\)This presentation follows Daniel and Shiamptanis (2012).
der of the paper simply as the interest rate. We derive restrictions below supporting this borrowing in equilibrium.

Define \( \alpha_t \) as default magnitude, according to

\[
\alpha_t = (1 - \delta_t) (1 + r_{t-1}) d_{t-1}.
\]

Using the above equations, unexpected default can be expressed as

\[
\alpha_t - E_{t-1} \alpha_t = -\delta_t (1 + r_{t-1}) d_{t-1} + (1 + r) d_{t-1}.
\]

Substituting \( \delta_t (1 + r_{t-1}) d_{t-1} \) into the equation for the evolution of debt yields

\[
d_t = (1 + r) d_{t-1} - s_t + E_{t-1} \alpha_t - \alpha_t.
\]

The expectation of default, \( E_{t-1} \alpha_t \), augments debt (by raising the domestic interest rate), whereas actual default, \( \alpha_t \), reduces debt. When the expectation of default equals actual default, there is no debt reduction because the interest rate fully adjusts to offset the future default.

Solving equation (2) forward for debt and imposing \( \lim_{T \to \infty} \frac{d_{t+T}}{(1+r)^T} = 0 \) yields an expression for the government’s intertemporal budget constraint,

\[
\sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^j (s_{t+j} + \alpha_{t+j} - E_{t+i-1} \alpha_{t+j}) = (1 + r) d_{t-1}.
\]

Equation (3) confirms that default provides revenue to augment the surplus only if it is unexpected.

### 2.1.2 Fiscal Solvency and the Effective Fiscal Limit

The definition of fiscal solvency with a fiscal limit has two components. First, the government’s expected intertemporal budget constraint must be satisfied. Second, expected future debt must remain below the fiscal limit. Together these requirements rule out an equilibrium in which debt rises forever because rising debt would eventually exceed the fiscal limit.\(^4\) We show that for solvency, government debt must remain on or below an

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3. The government’s no Ponzi game constraint rules out a positive value, while the no Ponzi game constraint for the household (or the aggregate of the remaining agents in the market) rules out a negative value.

4. Debt rising more slowly than the interest rate satisfies intertemporal budget balance.
effective fiscal limit, which we characterize.

Consider first expected intertemporal budget balance subject to debt not rising in the limit. Taking the expectation of equation (3) yields

$$\sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^j E_{t-1}s_{t+j} = (1 + r) d_{t-1}. \tag{3}$$

The expected intertemporal budget constraint is equivalent to the intertemporal budget constraint for the non-stochastic system. If the parameter governing the surplus-responsiveness to debt ($\gamma$) is independent of debt,$^5$ as implicitly assumed in equation (1), then expected intertemporal budget balance, subject to debt not rising in the limit, requires that the non-stochastic system be globally stable. This, in turn, requires that the characteristic roots, $\lambda_{1,2}$, given by

$$\lambda_{1,2} = \frac{1 + \beta + r - \gamma \pm \sqrt{(1 + \beta + r - \gamma)^2 - 4\beta (1 + r)}}{2},$$

both be inside the unit circle. We can simplify and express the requirement for stability as$^6$

$$\frac{\partial s_t}{\partial d_{t-1}} = \gamma > r (1 - \beta). \tag{4}$$

Stability requires that the surplus be sufficiently responsive to an increase in debt to bring the system to its long-run equilibrium, given by

$$d^* = r s^* = \frac{-c}{\gamma - r (1 - \beta)}, \tag{5}$$

where $d^*$ is the long-run value of debt and $s^*$ is the long-run value of the surplus.

The second requirement for solvency is that expected future values of debt must remain below the fiscal limit ($\bar{d}$), according to

$$E_{t-1}d_{t+i} \leq \bar{d} \text{ for all } i \geq 0.$$ 

Taking the current expectation of future values of debt, using equation (2) reveals that expectations do not depend on values for surplus shocks, expected default or actual default.

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$^5$We address a debt-dependent value for the responsiveness in the next section.

$^6$Other combinations of parameter values can be consistent with global stability but they violate magnitudes of coefficients compatible with the estimates in the data. Specifically, for $\beta (1 + r) < 1$, which always holds in the data, we need $r(1 - \beta) < \gamma < (1 + \beta)(2 + r)$. The second inequality always holds in the data, allowing us to focus on the first.
Expected future debt is the forward solution of the non-stochastic system of equations for debt. Given a value for \( s_{t-1} \), the largest value for \( d_{t-1} \), which is consistent with solvency, lies along the highest adjustment path for debt whose peak is less than or equal to the fiscal limit \((\bar{d})\). A larger value for debt would imply that debt would exceed its fiscal limit in the non-stochastic system. In this case, restraining debt below the fiscal limit, while continuing to follow the fiscal rule, would require systematic revenue from unexpected default and surplus shocks, an impossibility.

Therefore, if a country’s debt lies along a stable path whose peak is less than or equal to the fiscal limit, then the country is fiscally solvent. We define the highest such path for debt as the effective fiscal limit. Countries with debt on or below the effective fiscal limit are solvent, while those with debt above are insolvent.

We illustrate the effective fiscal limit using a phase diagram for the non-stochastic system in Figure 1. Subtracting the lagged value of the surplus from equation (1) and the lagged value of debt from equation (2) yields

\[
\Delta s_t = s_t - s_{t-1} = (\beta - 1) s_{t-1} + \gamma d_{t-1} + c = 0, \\
\Delta d_t = d_t - d_{t-1} = (r - \gamma) d_{t-1} - (\beta s_{t-1} + c) = 0.
\]

Setting the error to zero, the two equations for the phase diagram are given by

\[
d_{t-1}|(\Delta s = 0) = -\frac{c + (1 - \beta) s_{t-1}}{\gamma}, \\
d_{t-1}|(\Delta d = 0) = \frac{c + \beta s_{t-1}}{r - \gamma}.
\]

The \( \Delta s = 0 \) curve has a positive slope and, for \( \gamma > r \), the \( \Delta d = 0 \) has a negative slope.\(^7\) Arrows of motion for both curves point toward each curve, confirming a globally stable model.

In the diagram, the fiscal limit is the horizontal line at \( \bar{d} \). Point E represents long-run equilibrium values for debt and the surplus, given by equations (5). Adjustment paths AE, FE, and BE reflect paths for the surplus and debt toward the long-run equilibrium when the interest takes on its risk free value and there are no shocks.

\(^7\)When \( \Delta d = 0 \) has a positive slope, stability requires that it be steeper than \( \Delta s = 0 \), implying a similar pattern for arrows of motion.
A critical projection is the one for which the peak value of debt is the fiscal limit, $\bar{d}$, labeled FE. The effective fiscal limit is the upward-sloping portion of FE until it peaks at $\bar{d}$, and is $\bar{d}$ thereafter, the path labeled FGH. Beginning at any position along this path, expected future debt will reach the long-run without exceeding its fiscal limit, $\bar{d}$. And beginning at any point above, debt projections exceed the fiscal limit in transit to their long-run equilibrium values.

In summary, when a government faces a fiscal limit, solvency requires that debt be below the effective fiscal limit, defined as the highest stable path for debt whose peak is less than or equal to the fiscal limit.

### 2.1.3 Equilibrium Borrowing and the Effective Fiscal Limit

In this section, we derive restrictions allowing government borrowing in equilibrium. We show that for borrowing in equilibrium to be possible that government debt must lie below the effective fiscal limit. Equivalently, the government must be fiscally solvent.

We assume that the country sets post-default debt as the largest value of debt consistent with solvency, which is the effective fiscal limit. This assumption is consistent with our assumption that the government pays based on ability. We express the effective fiscal limit at a point in time as a value for debt which is linear in the surplus

\[
\hat{d}_t = \theta + \phi s_t \leq \bar{d}, \quad \theta \geq 0; \ \phi \geq 0, \tag{6}
\]
and view the values for $\theta$ and $\phi$ as time-varying linearizations about $t-1$ values for the surplus and debt. We are using equation (6) as an approximation of the effective fiscal limit FGH in Figure 1 at $t-1$ values for the debt and surplus.

When an unexpected shock sends debt above its effective fiscal limit, we assume that the government defaults. The magnitude of default returns debt to the effective fiscal limit, the highest value of debt consistent with solvency. Let $\Omega_t$ be the difference between the effective fiscal limit on debt, equation (6), and its current value, equation (2), given by

$$\Omega_t = \hat{d}_t - d_t = \theta + (1 + \phi) s_t - (1 + r) d_{t-1} - E_{t-1} \alpha_t + \alpha_t.$$  

Define the shadow value of default ($\tilde{\alpha}_t$) is the value of $\alpha_t$ which sets $\Omega_t = 0$,

$$\tilde{\alpha}_t = - [\theta + (1 + \phi) s_t - (1 + r) d_{t-1} - E_{t-1} \alpha_t].$$  

(7)

When the shadow value is positive, default equal to the shadow value sets $\Omega_t = 0$ and restores solvency. When the shadow value is negative, there is no default.

Now, solve for the equilibrium value for expected default. Let $x_{t-1}$ be difference between the projections for $\hat{d}_t$ and $d_t$ at the risk-free interest rate from time $t-1$. Substituting for the surplus, using equation (1), yields

$$x_{t-1} = \theta + (1 + \phi) \left[ (c + \beta s_{t-1}) - (1 + r - \gamma (1 + \phi)) d_{t-1} \right].$$

Rewrite the shadow value of default as

$$\tilde{\alpha}_t = -x_{t-1} + E_{t-1} \alpha_t - (1 + \phi) \epsilon_t,$$  

(8)

allowing $\Omega_t$ to be expressed as

$$\Omega_t = \alpha_t - \tilde{\alpha}_t.$$

Define $\epsilon_t^*$ as the critical value of the surplus shock, above which the country does not default next period. Using this definition, the expectation of default is

$$E_{t-1} \alpha_t = \int_{-\epsilon_t^*}^{\epsilon_t^*} \tilde{\alpha}_t f (\epsilon) d\epsilon = \int_{-\epsilon_t}^{\epsilon_t} [-x_{t-1} + E_{t-1} \alpha_t - (1 + \phi) \epsilon_t] f (\epsilon) d\epsilon$$  

$$= [-x_{t-1} + E_{t-1} \alpha_t] F (\epsilon_t^*) - (1 + \phi) \int_{-\epsilon_t}^{\epsilon_t^*} \epsilon_t f (\epsilon) d\epsilon.$$  

(9)
Continuing to solve by collecting terms on the expectation of default yields

$$E_{t-1}\alpha_t [1 - F(\epsilon_t^*)] = -x_{t-1}F(\epsilon_t^*) - (1 + \phi) \int_{-\bar{\epsilon}}^{\epsilon^*} \epsilon_t f(\epsilon) \, d\epsilon.$$ 

Substituting into equation (8) for the shadow value of default yields

$$\bar{\alpha}_t [1 - F(\epsilon_t^*)] = -[x_{t-1} + (1 + \phi) \epsilon_t] [1 - F(\epsilon_t^*)] - x_{t-1}F(\epsilon_t^*) - (1 + \phi) \int_{-\bar{\epsilon}}^{\epsilon^*} \epsilon_t f(\epsilon) \, d\epsilon$$

$$= -x_{t-1} - (1 + \phi) \left\{ \epsilon_t [1 - F(\epsilon_t^*)] + \int_{-\bar{\epsilon}}^{\epsilon^*} \epsilon_t f(\epsilon) \, d\epsilon \right\}.$$ 

Recall that default occurs when the shadow value is positive and does not occur when the shadow value is negative. Therefore, the critical value of the shock is the value of $\epsilon_t = \epsilon_t^* (-\bar{\epsilon} \leq \epsilon_t \leq \bar{\epsilon})$ which sets the above equation to zero.

There is a solution for $\epsilon_t^*$ iff $x_{t-1} \geq 0$. For $x_{t-1} = (1 + \phi) \bar{\epsilon}$, the critical value of the shock ($\epsilon_t^*$) equals its lower support ($-\bar{\epsilon}$). As $x_{t-1}$ falls, $\epsilon_t^*$ rises, reaching its peak at $\bar{\epsilon}$ once $x_{t-1} = 0$. For negative values of $x_{t-1}$, even the upper support ($\bar{\epsilon}$) does not satisfy the equation because at the upper support, the left-hand side of the equation is zero and the right-hand side is positive. Therefore, existence of an equilibrium value for expected default requires that $x_{t-1} \geq 0$.

Consider why non-negative $x_{t-1}$ is necessary for equilibrium. When $x_{t-1} \geq 0$, there are some realization(s) of the shock for which default does not occur. However, unless $x_{t-1} > (1 + \phi) \bar{\epsilon}$, there are realizations of the shock for which default does occur, implying that the expectation of default is positive. Positive expectations of default imply that, in the absence of default, debt could be expected to travel above its post-default value. However, the key for an equilibrium to exist is that there is at least one value of the shock for which this would not occur, as guaranteed by $x_{t-1} \geq 0$. Alternatively, when $x_{t-1} < 0$, default would occur for all realizations of the shock, implying that the probability of debt traveling above its post-default value is one. Therefore, $x_{t-1} < 0$ is not consistent with equilibrium.\(^8\)

Since $x_{t-1}$ is the difference between one-period ahead projections for the post-default value of debt ($\hat{d}_{t+1}$) and the actual value debt ($d_t$) at the risk-free rate, an equilibrium with lending can exist only when the projection for debt lies on or below the projection for the effective fiscal limit, governing the post-default value of debt. And since debt projections do not intersect, a positive distance between the effective fiscal limit and debt

\(^8\)Default would have occurred in period $t-1$, to assure that $x_{t-1} = 0$. 

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projected one-period ahead requires a positive distance today. Therefore, an equilibrium with expectations of default exists, when debt is on or below its effective fiscal limit.

To summarize, a country can borrow when debt is on or below the effective fiscal limit, given by the post-default value of debt. Since the post-default value of debt is the largest value of debt consistent with solvency, post-default debt is the effective fiscal limit. Therefore, when debt remains on or below the effective fiscal limit, the government is solvent and can borrow.

2.2 Dynamics with Fixed Responsiveness

We can use the phase diagrams for the non-stochastic system to understand how default keeps the system on or below the effective fiscal limit. Using Figure 1, consider an economy for which recent fiscal shocks have sent it to point A along the adjustment path AE. Point A is above the effective fiscal limit FG.\(^9\) Equivalently, the peak of the debt projection from point A exceeds the fiscal limit, \(\bar{d}\). Therefore, this debt-surplus pair represents a position of insolvency. An insolvent government cannot borrow to allow debt to rise along the adjustment path AE.

Alternatively, assume that this economy begins at point B. Points A and B both have identical initial debt, but point B has a higher surplus. Point B satisfies the solvency requirement, but there could be risk for insolvency depending on its proximity to the effective fiscal limit. If point B is sufficiently below the effective fiscal limit, implying that its one-period projections \((x_{t-1} > 0)\) are low enough that the probability of a shock sending it above in the next period is zero, then the interest rate initially takes on its risk-free value. From this position, the economy is expected to travel next period along the path BE toward its long-run equilibrium. However, if point B is not sufficiently below the effective fiscal limit, then the current interest rate includes a default premium, implying that, in the absence of default, debt is expected to rise more quickly than indicated by the BE path. In this case, an adverse shock next period, together with the system dynamics, could send the economy to a point beyond the effective fiscal limit FG. An economy beginning at point B is solvent, but depending on the distance to FG, the economy could face risk of insolvency in the next period or in the future. The initial values for both debt and the surplus play a critical role in determining solvency. Debt is identical at points A and B, but point A is fiscally insolvent, while point B is solvent with a risk of insolvency.

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\(^9\)One-period projections from point A \((x_{t-1} < 0)\) are also above the fiscal limit as claimed.
The presence of fiscal limits implies that the value of \( \gamma \), which represents the responsiveness of the surplus to debt, takes on a greater importance than its role in guaranteeing stability. In Figure 2, a larger responsiveness rotates the \( \Delta d = 0 \) curve counterclockwise and the \( \Delta s = 0 \) curve clockwise, reducing the area below the \( \Delta d = 0 \), in which debt is rising in favor of increasing the area above the \( \Delta d = 0 \), in which debt is falling. A large enough increase in \( \gamma \) could imply that an economy beginning at point A is fiscally solvent, as illustrated in Figure 2. With this different responsiveness, the projection for debt never rises above the fiscal limit, \( \tilde{d} \), along its trajectory toward the long-run equilibrium. Point A remains risky due to shocks, but higher responsiveness reduces the probability of insolvency below unity.

### 2.3 Time-varying Responsiveness

Ghosh et al. (2013) consider the possibility that the value for the responsiveness is not constant but is decreasing, at an increasing rate, in debt. In this case, the system could be stable for some values of debt and \( \gamma \), but unstable for higher values of debt and the associated lower values for \( \gamma \).
2.3.1 Cubic Model

Ghosh et al. (2013) estimate a large panel model of 23 advanced economies using data during the period 1970-2007. They utilize a cubic fiscal reaction function, which allows the surplus to respond to powers of debt given by

\[ s_t = c + \gamma_0^p d_{t-1} + \gamma_1^p d_{t-1}^2 + \gamma_2^p d_{t-1}^3 + \epsilon_t. \]

Responsiveness is the derivative of the surplus with respect to debt, explicitly

\[ \frac{\partial s_t}{\partial d_{t-1}} = \gamma_0^p + 2\gamma_1^p d_{t-1} + 3\gamma_2^p d_{t-1}^2. \]

They find that the responsiveness is negative at low levels of debt \((\gamma_0^p < 0)\). It becomes positive as debt rises \((\gamma_1^p > 0)\), and becomes negative again at high levels of debt \((\gamma_2^p < 0)\). Ghosh et al. (2013) label the fall in responsiveness at high levels of debt \((\gamma_2^p < 0)\) fiscal fatigue. Their cubic specification, together with the estimated negative value for \(\gamma_2^p\), implies that as debt increases from observed levels, there is always some level of debt, possibly well-outside the observed values in the data, for which responsiveness fails to satisfy the stability requirement.\(^{10}\) Therefore, for any country with \(\gamma_2^p < 0\), there is a value of debt beyond which the system becomes unstable. They refer to this value as the non-stochastic debt limit. For values of debt which begin below the debt limit, projections at the risk-free rate send debt toward a long-run equilibrium value. For values above, the debt projection is explosive. As long as the debt limit is below the fiscal limit, then the debt limit becomes the effective fiscal limit. In their specification, the limit is a point for debt, rather than a path for the debt-surplus pair, because the surplus does not respond to its own lag.

A crucial implication of the Ghosh et al. (2013) approach is that inference about the existence of the debt limit can be made, if and only if \(\gamma_2^p\) is negative. A positive estimate for \(\gamma_2^p\) fails to provide any information about the debt limit. Additionally, since responsiveness falls only at very high levels of debt, the authors rely on observations from a few high-debt countries to obtain estimates of \(\gamma_2^p\).\(^{11}\)

Finally, the estimated parameter values \((\gamma_0^p, \gamma_1^p, \gamma_2^p)\), together with individual-country interest rates, can imply that the cubic model is unstable for all values of debt, such that

\(^{10}\) With \(\beta = 0\) in their specification, global stability, equation (4), requires \(\frac{\partial s_t}{\partial d_{t-1}} = \gamma_0^p + 2\gamma_1^p d_{t-1} + 3\gamma_2^p d_{t-1}^2 > r.\)

\(^{11}\) Ghosh et al (2013) do provide some sensitivity analysis which they use to justify their assumption of identical responsiveness across countries. Our estimates reveal very different behavior across countries.
debt is never expected to reach a long-run equilibrium value. For these countries, if debt is explosive, agents should have refused to lend because they expected debt to exceed any fiscal limit. Ghosh et al. (2013) obtain unstable systems for five of their twenty-three countries, including Greece, Italy, and Portugal, for some of their assumptions about interest rates. If these are the correct interest rates, then the inference is that these countries should have experienced crises years ago. For other interest rates, they do obtain stable equilibria for these countries. However, the implied fiscal limit of 196.5% of GDP for Greece and 191.6% of GDP for Portugal are much too large, given that both countries lost access to markets with much lower debt levels. There seem to be no interest rate measures for which Greece and Portugal fit their experience of maintaining access to financial markets until after the world-wide financial crisis and then losing access with debt much lower than the estimated fiscal limits.

2.3.2 Threshold Model

An alternative way to model the nonlinear responsiveness as a function of debt is to allow the responsiveness to change as debt crosses a threshold. The threshold model differs from the cubic model in two respects. First, the threshold model does not make inferences about how responsiveness changes for values of debt outside the sample. The responsiveness does not continue to weaken as debt rises unless there is another threshold at higher debt with an even lower responsiveness. Second, a weaker responsiveness at high levels of debt yields information about the debt limit only if the threshold model fails to satisfy stability at these debt levels.

We consider the following threshold model, where the surplus responsiveness differs depending on whether debt has crossed a threshold. To avoid the assumption that countries have identical responsiveness to debt, we estimate individual-country fiscal rules. The surplus equation is given by

\[ s_t = c + \beta s_{t-1} + \gamma_0^h d_{t-1} + \gamma_1^h \left( d_{t-1} - \tilde{d}_1 \right) + \gamma_2^h \left( d_{t-1} - \tilde{d}_2 \right) + \epsilon_t. \]  

(11)

The terms \( \tilde{d}_1 \) and \( \tilde{d}_2 \) represent increasingly higher threshold values of debt. The coefficients \( \gamma_1^h \) and \( \gamma_2^h \) are non-zero only if the respective terms in parentheses are positive, and capture the changes in the surplus responsiveness as debt crosses the threshold levels \( \tilde{d}_1 \) and \( \tilde{d}_2 \), respectively. Responsiveness depends on the value of debt according to
\[
\frac{\partial s_t}{\partial d_{t-1}} = \begin{cases} 
\gamma_0^h & \text{for } d_{t-1} < \tilde{d}_1 \\
\gamma_0^h + \gamma_1^h & \text{for } \tilde{d}_1 < d_{t-1} < \tilde{d}_2 \\
\gamma_0^h + \gamma_1^h + \gamma_2^h & \text{for } d_{t-1} < \tilde{d}_2
\end{cases}
\]

Responsiveness is therefore a step function in the level of debt. The threshold model allows us to distinguish between the case in which the responsiveness weakens at high debt levels \((\gamma_2^h < 0)\) but remains large enough to imply stability \(\left(\frac{\partial s_t}{\partial d_{t-1}} = \gamma_0^h + \gamma_1^h + \gamma_2^h > r (1 - \beta)\right)\), and an alternative in which responsiveness weakens \((\gamma_2^h < 0)\) sufficiently to yield instability \(\left(\frac{\partial s_t}{\partial d_{t-1}} = \gamma_0^h + \gamma_1^h + \gamma_2^h < r (1 - \beta)\right)\). If the model fails the stability criterion at high values of debt, then the second threshold can be used to infer the effective fiscal limit on debt, as in the cubic model by Ghosh et al. (2013).

Figure 3 provides a phase diagram of the non-stochastic system for a country which fails the stability criterion above the second threshold \(\tilde{d}_2\), but satisfies it below \(\tilde{d}_2\). The time paths for the surplus and debt can be used to express the saddlepath relationship between debt and the surplus as

\[
\hat{d}_t = D + \left(\frac{\lambda_2 - \beta}{\gamma_0^h + \gamma_1^h + \gamma_2^h}\right) (s_t - rD),
\]

where \(D\) is the level of debt at the intersection point between the \(\Delta s = 0\) and \(\Delta d = 0\) curves above \(\tilde{d}_2\), given by

\[
D = \frac{-c + \gamma_1^h \tilde{d}_1 + \gamma_2^h \tilde{d}_2}{\gamma_0^h + \gamma_1^h + \gamma_2^h - r (1 - \beta)} > \tilde{d}_2,
\]

and \(\lambda_2\) is the stable root of the dynamic system, given by equations (11) and (2) for debt above the second threshold.

We can use the phase diagram of the non-stochastic system in Figure 3 to understand the fully stochastic system. The endogenous default premium accentuates the instability on and above the saddlepath. Along the saddlepath, the default premium implies that in the absence of default, debt is expected to rise more quickly than along the saddlepath such that the country avoids insolvency only for the largest possible surplus shock. However, since there is a value of the surplus shock for which the government is solvent, there is an equilibrium interest rate at which agents lend. For positions above the saddlepath, there is no equilibrium interest rate, implying insolvency. For positions below, in the presence of default, debt rises more quickly or declines more slowly than the arrows imply. Therefore, the saddlepath becomes the effective fiscal limit with \(\phi = \left(\frac{\lambda_2 - \beta}{\gamma_0^h + \gamma_1^h + \gamma_2^h}\right)\) and \(\theta = D (1 - \phi r)\).
There is risk in the neighborhood below the saddlepath, but the probability of insolvency remains less than unity and agents lend.

![Saddlepath Diagram](image)

**Figure 3: A country which fails the stability criterion**

Note that the saddlepath, and equivalently the effective fiscal limit, is above the threshold \( \tilde{d}_2 \) beyond which the system becomes unstable. Therefore, a country’s debt can cross the threshold \( \tilde{d}_2 \), at which responsiveness falls sufficiently to fail the stability criterion, without exceeding its effective fiscal limit. The country retains access to financial markets in this region, enabling estimation of the threshold and the associated saddlepath. Since agents are lending, we can estimate the parameters necessary to construct the saddlepath.

In a model with debt thresholds, a negative \( \gamma_2^h \) does not always provide evidence about the effective fiscal limit. Figure 4 provides a phase diagram for a country with a second negative threshold \( \gamma_2^h < 0 \), but for which the stability criterion is satisfied for all values of debt. Since there are no explosive regions, we cannot infer an effective fiscal limit on debt. Arrows of motion send the system toward the equilibrium, which is between the two threshold values. Even if debt is well above the second threshold \( \tilde{d}_2 \), projections send the economy to its long-run equilibrium. This region could have risk of default if debt is near the actual fiscal limit since expectations of default together with shocks could send the economy above the actual fiscal limit (not shown). However, as long as there are values of the shock for which default does not occur, there is an equilibrium interest...
rate, and therefore an equilibrium with borrowing.

![Graph](image)

Figure 4: A country which does not fail the stability criterion

3 Estimates of Fiscal Rules

We estimate the responsiveness of the surplus to debt for ten high-debt countries, Belgium, Canada, France, Greece, Italy, Japan, Portugal, Spain, UK and US using annual data during the period 1970-2007. We cut our sample in 2007 for two reasons. First, no country in our sample experienced a fiscal crisis prior to 2007, implying that countries were able to borrow to follow their fiscal rules over the sample. Second, we use data prior to the world-wide financial crisis (in sample data) to assess solvency risks after the
financial crisis (out of sample). Summary statistics are reported in Table 1.\footnote{The variables are from the OECD (Economic Outlook No. 96) and AMECO database. For $s_t$ we use the general government primary balances relative to GDP, for $d_t$ we use the gross public debt relative to GDP.}

<table>
<thead>
<tr>
<th></th>
<th>Debt as % of GDP</th>
<th>Surplus as % of GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Mean</td>
</tr>
<tr>
<td>Belgium</td>
<td>54.28</td>
<td>98.31</td>
</tr>
<tr>
<td>Canada</td>
<td>44.08</td>
<td>71.45</td>
</tr>
<tr>
<td>France</td>
<td>20.39</td>
<td>40.39</td>
</tr>
<tr>
<td>Greece</td>
<td>14.98</td>
<td>59.39</td>
</tr>
<tr>
<td>Italy</td>
<td>35.71</td>
<td>82.27</td>
</tr>
<tr>
<td>Japan</td>
<td>69.95</td>
<td>79.78</td>
</tr>
<tr>
<td>Portugal</td>
<td>13.30</td>
<td>45.66</td>
</tr>
<tr>
<td>Spain</td>
<td>11.50</td>
<td>36.51</td>
</tr>
<tr>
<td>UK</td>
<td>31.36</td>
<td>48.20</td>
</tr>
<tr>
<td>US</td>
<td>42.63</td>
<td>55.85</td>
</tr>
</tbody>
</table>

Equation (1) posits a constant linear response of the surplus to debt, but Bohn (1998, 2008) and Ghosh et al. (2013) find evidence in favor of a time-varying responsiveness. We consider two alternative sources for the time-variation. The first follows Bohn’s (1998, 2008) and Ghosh et al.’s (2013) approach, which allows responsiveness to vary with debt. We estimate a threshold model to deal with possible problems raised by the cubic specification. When responsiveness varies with debt, parameters can take on values which allow inference about the values of the effective fiscal limit. For the second, we present and test the hypothesis that responsiveness varies with the interest rate. In this approach, estimates provide no evidence about the effective fiscal limit, although they do allow inference on the long-run value of debt, which we argue must be below the fiscal limit.

3.1 Surplus Responsiveness Changes with Debt

Before we proceed with the estimation of the fiscal rule parameters $(\beta, \gamma^h_0, \gamma^h_1, \gamma^h_2)$ and the threshold values of debt $\left(\tilde{d}_1, \tilde{d}_2\right)$, we verify that there is evidence for threshold effects in the data. We use the heteroskedasticity-consistent Lagrange multiplier (LM) test of Hansen (1996), which tests the null hypothesis of no thresholds against a single threshold.
Additionally, we employ the double maximum tests, $UD_{\text{max}}$ and $WD_{\text{max}}$, of Bai and Perron (1998, 2003), which do not require the pre-specification of the number of thresholds in the alternative hypothesis.\textsuperscript{13} They test the null hypothesis of no thresholds against an unknown number of thresholds, given some maximum number. We allow for up to three thresholds and the results are reported in Table 2. For all countries except Portugal, the tests suggest the presence of thresholds in debt.

<table>
<thead>
<tr>
<th></th>
<th>LM</th>
<th>$UD_{\text{max}}$</th>
<th>$WD_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>10.72*</td>
<td>16.80†</td>
<td>24.46‡</td>
</tr>
<tr>
<td>Canada</td>
<td>12.01†</td>
<td>14.35†</td>
<td>20.01‡</td>
</tr>
<tr>
<td>France</td>
<td>13.89‡</td>
<td>14.39‡</td>
<td>14.39‡</td>
</tr>
<tr>
<td>Greece</td>
<td>11.13*</td>
<td>59.60‡</td>
<td>86.78‡</td>
</tr>
<tr>
<td>Italy</td>
<td>11.08*</td>
<td>23.49‡</td>
<td>34.20‡</td>
</tr>
<tr>
<td>Japan</td>
<td>8.65</td>
<td>12.20†</td>
<td>16.08‡</td>
</tr>
<tr>
<td>Portugal</td>
<td>6.43</td>
<td>6.39</td>
<td>7.04</td>
</tr>
<tr>
<td>Spain</td>
<td>11.47†</td>
<td>18.48‡</td>
<td>18.48‡</td>
</tr>
<tr>
<td>UK</td>
<td>12.20†</td>
<td>14.90‡</td>
<td>14.90‡</td>
</tr>
<tr>
<td>US</td>
<td>16.13‡</td>
<td>11.15*</td>
<td>12.23*</td>
</tr>
</tbody>
</table>

Note: The *, † and ‡ indicate statistical significance at the 10, 5 and 1 percent level, respectively.

Given that the fiscal policy rules appear to have threshold effects, we estimate the threshold values of debt $\tilde{d} = [\tilde{d}_1 \ \tilde{d}_2]'$. We follow Hansen (2000) who recommends estimation of $\tilde{d}$ by minimizing the sum of squared errors ($SSE$) of equation (11), $\tilde{d} = \arg \min_{\tilde{d}} SSE(\tilde{d})$. Hansen’s procedure estimates one threshold at a time. The first threshold level is chosen as the one which allows the greatest reduction in the SSE. To identify the second threshold, we follow the sequential approach by Bai and Perron (1998, 2003). We test the null hypothesis of $l$ thresholds against the alternative of $l + 1$ thresholds using the sup $F_T(l + 1|l)$ statistic. If the overall minimum value of the SSE with $l + 1$ thresholds is sufficiently smaller than the SSE from the model with $l$ thresholds, then the null is rejected and the model with $l + 1$ thresholds is chosen and the threshold selected is associated with this overall minimum SSE. This process is repeated by increasing $l$ sequentially.

\textsuperscript{13}Bai and Perron (2003) show that their structural changepoint tests can be applied to threshold models by first sorting the data based on the threshold variable and then applying their structural changepoint model.
until the test fails to reject the null hypothesis.\footnote{Bai (1997) showed that it is possible to consistently estimate all breaks sequentially one at a time.} Table 3 contains the estimates of the debt thresholds, together with the 95% confidence intervals\footnote{Following Hansen (2000), the 95% confidence region is set for values where the likelihood ratio statistic is less than the 95% critical value of 7.35. The likelihood ratio statistic is evaluated over actual values of debt after they have been sorted from smallest to largest. This often yields asymmetric confidence intervals because the sorted data is discrete with uneven increments.}, and dates for which debt exceeded each threshold.

### Table 3 Debt Thresholds as % of GDP

<table>
<thead>
<tr>
<th>Country</th>
<th>First threshold</th>
<th>Second threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d_1$</td>
<td>Interval Dates</td>
</tr>
<tr>
<td>France</td>
<td>39.88 [33.56-46.23]</td>
<td>1992-2007</td>
</tr>
<tr>
<td>Italy</td>
<td>81.91 [66.79-94.84]</td>
<td>1986-2007</td>
</tr>
<tr>
<td>Portugal</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>UK</td>
<td>55.66 [50.16-58.50]</td>
<td>1970-1978</td>
</tr>
<tr>
<td>US</td>
<td>59.29 [58.75-61.54]</td>
<td>1987;1989-1997;2004-2007</td>
</tr>
</tbody>
</table>

Note that thresholds differ widely across countries, both in their numbers and their magnitudes. Portugal has no thresholds; Canada, France, Italy, the UK and the US, each have one; Belgium, Greece, Japan and Spain, each have two. Values for the first and second thresholds are extremely different, with first-threshold values ranging from a low of 29.56% of GDP for Spain to a high of 81.91% of GDP for Italy. Second thresholds exist for only four countries, and they are also very different, with Spain again having the low value of 42.32% of GDP and Belgium the high at 104.88%. For most countries, the highest debt threshold occurs in the middle of the sample, while for the UK, it occurs in the early portion of the sample. We find no evidence for three thresholds.

Next, consider estimation of the fiscal policy parameters $(\beta, \gamma^h_0, \gamma^h_1, \gamma^h_2)$. We use the threshold estimates from Table 3 to construct the spline terms in equation (11). After constructing the spline terms, we estimate the policy parameters using least squares as recommended by Hansen (2000) and Bai and Perron (1998, 2003). We use White robust standard errors to address potential concerns about heteroskedasticity. Following
Bohn (1998, 2008), we include the output gap ($\tilde{y}_t$) and temporary outlays in government spending ($\tilde{g}_t$), where $\delta_1$ and $\delta_1$ are the parameters on $\tilde{y}_t$ and $\tilde{g}_t$, respectively. \footnote{For $\tilde{y}_t$ we use the economy’s output gap and for $\tilde{g}_t$ we use the cyclical component of the log real government consumption expenditure obtained from the Hodrick-Prescott filter with the smoothing parameter set at 100, as in Mendoza and Ostry (2008).}

<table>
<thead>
<tr>
<th></th>
<th>Belgium</th>
<th>Canada</th>
<th>France</th>
<th>Greece</th>
<th>Italy</th>
<th>Japan</th>
<th>Portugal</th>
<th>Spain</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>8.764</td>
<td>-0.859</td>
<td>0.333</td>
<td>0.317</td>
<td>-4.638$\dagger$</td>
<td>-3.501$\dagger$</td>
<td>-1.083$\dagger$</td>
<td>-0.997$*$</td>
<td>-3.154$\dagger$</td>
<td>-6.155$\dagger$</td>
</tr>
<tr>
<td></td>
<td>(5.375)</td>
<td>(1.383)</td>
<td>(0.555)</td>
<td>(0.438)</td>
<td>(1.105)</td>
<td>(1.129)</td>
<td>(0.610)</td>
<td>(0.590)</td>
<td>(1.518)</td>
<td>(1.968)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.599$\dagger$</td>
<td>0.606$\dagger$</td>
<td>0.202$*$</td>
<td>0.489$\dagger$</td>
<td>0.291$\dagger$</td>
<td>0.393$*$</td>
<td>0.498$\dagger$</td>
<td>0.685$\dagger$</td>
<td>0.630$\dagger$</td>
<td>0.550$\dagger$</td>
</tr>
<tr>
<td></td>
<td>(0.077)</td>
<td>(0.062)</td>
<td>(0.123)</td>
<td>(0.138)</td>
<td>(0.127)</td>
<td>(0.222)</td>
<td>(0.137)</td>
<td>(0.101)</td>
<td>(0.140)</td>
<td>(0.081)</td>
</tr>
<tr>
<td>$\gamma_0^h$</td>
<td>-0.172$*$</td>
<td>-0.004</td>
<td>-0.035$\dagger$</td>
<td>-0.040$\dagger$</td>
<td>0.028$\dagger$</td>
<td>0.055$\dagger$</td>
<td>0.026$\dagger$</td>
<td>-0.036</td>
<td>0.073$\dagger$</td>
<td>0.109$\dagger$</td>
</tr>
<tr>
<td></td>
<td>(0.094)</td>
<td>(0.027)</td>
<td>(0.016)</td>
<td>(0.019)</td>
<td>(0.013)</td>
<td>(0.021)</td>
<td>(0.013)</td>
<td>(0.034)</td>
<td>(0.033)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>$\gamma_1^h$</td>
<td>0.269$\dagger$</td>
<td>0.073$*$</td>
<td>0.058$\dagger$</td>
<td>0.099$\dagger$</td>
<td>0.134$\dagger$</td>
<td>-0.262$\dagger$</td>
<td>0.180$\dagger$</td>
<td>-0.211$\dagger$</td>
<td>-0.146$*$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.117)</td>
<td>(0.040)</td>
<td>(0.028)</td>
<td>(0.045)</td>
<td>(0.039)</td>
<td>(0.086)</td>
<td>(0.070)</td>
<td>(0.079)</td>
<td>(0.080)</td>
<td></td>
</tr>
<tr>
<td>$\gamma_2^h$</td>
<td>-0.082$\dagger$</td>
<td>-0.186$*$</td>
<td>0.261$\dagger$</td>
<td>-0.108$\dagger$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.099)</td>
<td>(0.078)</td>
<td>(0.055)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.445$\dagger$</td>
<td>0.403$\dagger$</td>
<td>0.228$*$</td>
<td>0.099</td>
<td>0.192</td>
<td>0.180</td>
<td>0.055</td>
<td>0.291$\dagger$</td>
<td>0.138$\dagger$</td>
<td>0.229$\dagger$</td>
</tr>
<tr>
<td></td>
<td>(0.203)</td>
<td>(0.086)</td>
<td>(0.120)</td>
<td>(0.061)</td>
<td>(0.174)</td>
<td>(0.125)</td>
<td>(0.049)</td>
<td>(0.104)</td>
<td>(0.058)</td>
<td>(0.079)</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-0.403$\dagger$</td>
<td>-0.354$\dagger$</td>
<td>-0.500$\dagger$</td>
<td>-0.148$\dagger$</td>
<td>-0.022</td>
<td>-0.341$\dagger$</td>
<td>-0.036</td>
<td>-0.250$\dagger$</td>
<td>-0.559$\dagger$</td>
<td>-0.528$*$</td>
</tr>
<tr>
<td></td>
<td>(0.211)</td>
<td>(0.064)</td>
<td>(0.097)</td>
<td>(0.055)</td>
<td>(0.110)</td>
<td>(0.092)</td>
<td>(0.091)</td>
<td>(0.070)</td>
<td>(0.168)</td>
<td>(0.103)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.895</td>
<td>0.944</td>
<td>0.639</td>
<td>0.714</td>
<td>0.918</td>
<td>0.791</td>
<td>0.353</td>
<td>0.918</td>
<td>0.730</td>
<td>0.764</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.343</td>
<td>0.823</td>
<td>0.661</td>
<td>1.03</td>
<td>1.049</td>
<td>1.396</td>
<td>1.462</td>
<td>0.862</td>
<td>1.274</td>
<td>0.865</td>
</tr>
</tbody>
</table>

Table 4 reveals that all countries respond significantly positively to the lagged surplus, a behavior important in dynamics, but normally omitted from surplus regressions. \footnote{Examples include Ghosh et al. (2013), Mendoza and Ostry (2008), Bohn (1998, 2007, 2008).} However, the response of the surplus to debt at different thresholds is highly varied. Only half of the countries respond significantly and positively at low levels of debt. Others respond significantly and negatively, or insignificantly. As debt rises above subsequent thresholds, changes in responsiveness are heterogeneous across countries. Consider, first, the countries with two thresholds. With debt greater than the first threshold but less than the second threshold, Japan is least responsive, while Belgium, Greece, and Spain are most responsive. For countries with a single threshold, Canada, France, and Italy be-
come more responsive as debt rises above the threshold, while the UK and the US become less responsive. And Portugal has no thresholds at all, implying that responsiveness does not change as debt rises. Five of the ten countries\textsuperscript{18} exhibit the reduced responsiveness to debt which Ghosh et al. (2013) find in their cubic panel model.

If the total responsiveness violates the stability criterion \( \left( \frac{\partial st}{\partial ds_{t-1}} = \gamma_0^h + \gamma_1^h + \gamma_2^h < r (1 - \beta) \right) \) as debt rises above its final threshold, we can infer the value for the effective fiscal limit as in Ghosh et al. (2013)\textsuperscript{19}. Although Japan has negative responsiveness when debt crosses the first threshold, once debt becomes high enough to cross the second threshold, responsiveness rises sufficiently to prevent instability and the associated explosive behavior of debt. For Belgium and Spain, responsiveness weakens for debt above the second threshold \( (\gamma_2^h < 0) \), but both countries continue to satisfy the stability criterion.\textsuperscript{20} For the UK and the US, responsiveness becomes negative for debt above the single threshold and for Greece responsiveness is negative for debt above its second threshold.\textsuperscript{21} Therefore, the threshold model can provide information about the effective fiscal limit for three of our ten countries, specifically for Greece, the UK and the US.

The post-default value of debt, and equivalently, the effective fiscal limit is the saddlepath in the non-stochastic system. We construct the saddlepath using equation (12), together with the fiscal policy parameters reported in Table 4, and an interest rate for each country. The interest rate we use is the average of the growth-adjusted interest rates over the period for which responsiveness violates the stability criterion. These interest rates could be too high if they include default premia, but these countries were not cited as in fiscal distress over this period. The phase diagram for values of debt beyond the second threshold for Greece and beyond the first threshold for the UK and the US, together with the saddlepath and the actual values for the surplus and debt over this period, are plotted in the left panel of Figure 5.

\textsuperscript{18}These countries are Belgium, Greece, Spain, the UK and the US.

\textsuperscript{19}We use two different interest rates with the same inference on stability. We use the average growth-adjusted interest rate over the full sample period and over the period for which debt exceeds the negative threshold.

\textsuperscript{20}The threshold model does not require responsiveness to continue to weaken as debt increases. Therefore, if the model remains stable at the threshold, the inference is that it is stable at higher levels of debt, in contrast to the cubic model.

\textsuperscript{21}The combined negative responsiveness for the UK is statistically significant at the 1\% level, for Greece at the 10\% level, and for the US is not statistically different from zero.
The diagrams provide tests of whether our estimates of effective fiscal limits for these three countries are consistent with financial market behavior over the period. The effective fiscal limit is the saddlepath, above which there is no interest rate which can compensate both for the expectation of default and the excess of debt above its post-default value.
Therefore, countries should have lost access to financial markets in this region. Since none of these countries lost access to financial markets over this period, we should find that all debt-surplus pairs lie below each respective saddlepath.

Both Greece and the US fail this test. Greece should have lost access to markets as early as 2005, and the US should not have had access over a large portion of the 1990’s and the 2000’s. The UK has no debt-surplus pairs above its saddlepath, thereby passing the test. However, the position of each saddlepath is highly dependent on the value of the interest rate we use to construct it. In the right panel of Figure 5, we show that slight reductions in interest rates raise the saddlepath sufficiently to place all debt-surplus pairs below the saddlepath for both Greece and the US. This implies that rejection of our estimate of the effective fiscal limit as the saddlepath is weak. However, if we were to add the data after the financial crisis, with high values of debt and low values for the surplus, then we find that both the US and the UK have debt-surplus pairs above their saddlepaths, thereby failing the test.

We conclude that a model in which surplus responsiveness varies with debt does not provide a reliable estimate of a country’s effective fiscal limit for several reasons. First, for the threshold model to provide any evidence on the effective fiscal limit requires that the estimate for total responsiveness at high levels of debt violate the stability condition. Since this occurs in only three of the ten countries, our ability to estimate the effective fiscal limit is highly restricted. And since we find that the surplus-responsiveness to debt varies so widely over countries, we do not want to use behavior by some countries to infer effective fiscal limits for others. Second, even for the countries with low enough responsiveness to violate the stability criterion, our estimates of effective fiscal limits appear too low, especially in light of behavior following the financial crisis. Finally, the highly varied surplus responsiveness to debt across countries, as debt rises, together with questionable values of estimated fiscal limits for the countries that do exhibit fiscal fatigue, leads us to question the assumption that surplus responsiveness is actually changing with debt. If surplus responsiveness is changing with some variable whose correlation with debt varies across countries, then our model is misspecified.

The alternative variable we consider as a possible determinate of responsiveness is the interest rate. The level of debt could pick up interest-rate behavior because at the beginning of the sample, for most countries, both debt and interest rates were low, in the middle of the sample both debt and interest rates were high, and at the end of the sample the behavior of debt and interest rates diverged. Interest rates fell for most countries, while debt for some countries rose and for others fell. In the next section, we investigate
the possibility that responsiveness varies with the interest rate, and not explicitly with
the value of debt itself.

3.2 Surplus Responsiveness Changes with Interest Rate

Growth-adjusted interest rates for all countries have similar movements over our sam-
ple and take on a wide range of values. They are negative in the early part of the sample,
beginning with the 1970’s and continuing into the 1980’s. The negativity comes from high
inflation and growth compared to the nominal interest rate. In the mid-1980’s, interest
rates rise with a sharp increase in nominal interest rates, and they remain positive until
the late-1990’s for some countries and until the end of the sample for others. For all
countries except Japan, the interest rates are higher in the middle of the sample than
they are early or late. Japan’s interest rates rise throughout the sample. Fiscal risk is
affected by the value of the interest rate since a higher interest rate rotates the \( \Delta d = 0 \)
curve clockwise, widening the space in debt and the surplus for which debt is rising. A
rise in responsiveness reverses this rotation, offsetting this additional risk.

We view these periods in which interest rates took on very different values as different
interest rate regimes and test whether the responsiveness takes on different values in
different interest rate regimes. To identify different interest rate regimes for each country,
we test for multiple break points using the sequential procedure of Bai and Perron (1998,

### Table 5 Interest Rate Regimes

<table>
<thead>
<tr>
<th>Country</th>
<th>First Regime</th>
<th></th>
<th>Second Regime</th>
<th></th>
<th>Third Regime</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Period</td>
<td></td>
<td>Period</td>
<td></td>
<td>Period</td>
<td></td>
</tr>
</tbody>
</table>

26
We allow for up to 5 breaks and serial correlation in the errors. At the 5% significance level, we find three separate interest rate regimes for all countries except Italy, which has only two. The dates for each regime over the sample 1970-2007, together with the mean value of the growth-adjusted interest rates in each regime (percents), $r_i$, are given in Table 5.

Consider a model, which allows the constant and the responsiveness to depend on the interest rate regime according to

$$s_t = c_i + \beta s_{t-1} + \gamma_i^r d_{i,t-1} + \epsilon_t,$$

(14)

where $i$ denotes the distinct interest rate regimes and $\gamma_i^r$ represents the responsiveness in that particular regime, not the additional responsiveness in that regime, as in the threshold model. We use the break dates from Table 5 to construct dummy variables for each interest rate regime, and use them to estimate equation (14). Results are contained in Table 6. The interest rate model (equation 14) fits well, typically with adjusted $R^2$ measures slightly higher and standard errors slightly lower than for the threshold model (equation 11).

The results confirm our hypothesis that responsiveness increases in interest rates. We find that in the first sub-period, when interest rates are negative for all countries, responsiveness ($\gamma_1^r$) is never both positive and significantly different from zero. Therefore, in the first regime with negative interest rates, countries were either responding negatively to debt or were not responding at all. In the middle period when interest rates for all countries rise, responsiveness ($\gamma_2^r$) rises above zero for all countries and is statistically significant at the 1% percent level for most countries. In the third period when interest rates fall for all countries except Japan, responsiveness falls ($\gamma_3^r$) for all countries except Japan. For Japan, the interest rate rises and responsiveness rises. Therefore, for all countries, responsiveness increases systematically with the interest rate. This provides evidence that the reduced responsiveness by most countries at the end of the sample is due to reduced interest rates. This shows up as fiscal fatigue for countries whose debt rose at the end of the sample, and as contradictions to fiscal fatigue for countries whose debt fell at the end of the sample. The hypothesis that responsiveness varies positively with the interest rate tells a more consistent story across countries and across time than

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the hypothesis of fiscal fatigue, which holds for three countries but not for seven others.25

### Table 6 Responsiveness Varying with Interest Rate

<table>
<thead>
<tr>
<th></th>
<th>Belgium</th>
<th>Canada</th>
<th>France</th>
<th>Greece</th>
<th>Italy</th>
<th>Japan</th>
<th>Portugal</th>
<th>Spain</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>-2.534</td>
<td>-6.346</td>
<td>0.499</td>
<td>1.672</td>
<td>-5.241</td>
<td>-1.807</td>
<td>0.637</td>
<td>-4.982</td>
<td>6.233</td>
<td>-11.022</td>
</tr>
<tr>
<td></td>
<td>(6.601)</td>
<td>(1.882)</td>
<td>(0.382)</td>
<td>(1.960)</td>
<td>(5.019)</td>
<td>(0.799)</td>
<td>(2.535)</td>
<td>(2.555)</td>
<td>(5.981)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.767)</td>
<td>(0.559)</td>
<td>(0.517)</td>
<td>(1.700)</td>
<td>(1.837)</td>
<td>(1.086)</td>
<td>(2.633)</td>
<td>(1.272)</td>
<td>(1.464)</td>
<td>(2.460)</td>
</tr>
<tr>
<td></td>
<td>(2.663)</td>
<td>(1.124)</td>
<td>(4.067)</td>
<td>(7.032)</td>
<td>(3.563)</td>
<td>(2.228)</td>
<td>(1.096)</td>
<td>(3.508)</td>
<td>(2.120)</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.353</td>
<td>0.588</td>
<td>0.290</td>
<td>0.368</td>
<td>0.449</td>
<td>0.619</td>
<td>0.253</td>
<td>0.442</td>
<td>0.614</td>
<td>0.620</td>
</tr>
<tr>
<td></td>
<td>(0.136)</td>
<td>(0.065)</td>
<td>(0.138)</td>
<td>(0.122)</td>
<td>(0.130)</td>
<td>(0.131)</td>
<td>(0.142)</td>
<td>(0.140)</td>
<td>(0.096)</td>
<td></td>
</tr>
<tr>
<td>$\gamma_1^r$</td>
<td>0.011</td>
<td>0.113</td>
<td>-0.037</td>
<td>-0.110</td>
<td>0.055</td>
<td>0.041</td>
<td>-0.118</td>
<td>0.231</td>
<td>-0.104</td>
<td>0.211</td>
</tr>
<tr>
<td></td>
<td>(0.119)</td>
<td>(0.067)</td>
<td>(0.021)</td>
<td>(0.019)</td>
<td>(0.039)</td>
<td>(0.380)</td>
<td>(0.069)</td>
<td>(0.184)</td>
<td>(0.042)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>$\gamma_2^r$</td>
<td>0.101</td>
<td>0.077</td>
<td>0.075</td>
<td>0.067</td>
<td>0.084</td>
<td>0.029</td>
<td>0.085</td>
<td>0.097</td>
<td>0.160</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.006)</td>
<td>(0.010)</td>
<td>(0.020)</td>
<td>(0.020)</td>
<td>(0.016)</td>
<td>(0.052)</td>
<td>(0.032)</td>
<td>(0.046)</td>
<td></td>
</tr>
<tr>
<td>$\gamma_3^r$</td>
<td>0.070</td>
<td>0.051</td>
<td>0.055</td>
<td>0.057</td>
<td>0.070</td>
<td>0.025</td>
<td>0.035</td>
<td>0.077</td>
<td>0.076</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.013)</td>
<td>(0.064)</td>
<td>(0.070)</td>
<td>(0.024)</td>
<td>(0.039)</td>
<td>(0.020)</td>
<td>(0.082)</td>
<td>(0.034)</td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.336</td>
<td>0.346</td>
<td>0.334</td>
<td>0.013</td>
<td>0.164</td>
<td>0.002</td>
<td>0.089</td>
<td>0.288</td>
<td>0.137</td>
<td>0.305</td>
</tr>
<tr>
<td></td>
<td>(0.148)</td>
<td>(0.046)</td>
<td>(0.086)</td>
<td>(0.068)</td>
<td>(0.211)</td>
<td>(0.120)</td>
<td>(0.047)</td>
<td>(0.074)</td>
<td>(0.066)</td>
<td>(0.060)</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-0.223</td>
<td>-0.396</td>
<td>-0.559</td>
<td>-0.132</td>
<td>-0.153</td>
<td>-0.241</td>
<td>-0.002</td>
<td>-0.138</td>
<td>-0.519</td>
<td>-0.657</td>
</tr>
<tr>
<td></td>
<td>(0.227)</td>
<td>(0.049)</td>
<td>(0.147)</td>
<td>(0.073)</td>
<td>(0.124)</td>
<td>(0.229)</td>
<td>(0.083)</td>
<td>(0.082)</td>
<td>(0.199)</td>
<td>(0.119)</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.908</td>
<td>0.960</td>
<td>0.683</td>
<td>0.761</td>
<td>0.904</td>
<td>0.760</td>
<td>0.405</td>
<td>0.929</td>
<td>0.712</td>
<td>0.846</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.201</td>
<td>0.711</td>
<td>0.620</td>
<td>0.944</td>
<td>1.132</td>
<td>1.495</td>
<td>1.402</td>
<td>0.802</td>
<td>1.318</td>
<td>0.700</td>
</tr>
</tbody>
</table>

Note: The *, † and ‡ indicate statistical significance at the 10, 5 and 1 percent level, respectively.

We test whether the interest rate regimes are distinct with F-tests for the equality of the regimes. Table 7 reports the p-values of the F-tests. For all countries except the US, the early low-interest rate regime, labeled R1, is statistically different from the very highest one. The highest interest rate regime is R2 for all countries except Japan and is R3 for Japan. Additionally, for all countries except UK, the middle regime, which has maximum interest rates for all countries except Japan, is statistically different from final regime. There are two anomalies in the table. First, for the UK, the high interest rate

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25 We experimented with allowing responsiveness to change with a lag to a new interest-rate regime with similar results.
regime, labeled R2, is not statistically different from the later lower interest rate regime, labeled R3. This might be because the UK experienced the smallest fall in the interest rate between the two regimes. Second, for the US, R1 is not statistically different from R2 or from R3, yet, R2 is statistically different from R3. The problem seems to be that estimates in R1 are imprecise, implying that the R1 regime is not very different from either the R2 or R3 regimes.

<table>
<thead>
<tr>
<th></th>
<th>p(R1 = R2)</th>
<th>p(R2 = R3)</th>
<th>p(R1 = R3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>0.089</td>
<td>0.007</td>
<td>0.508</td>
</tr>
<tr>
<td>Canada</td>
<td>0.000</td>
<td>0.104</td>
<td>0.027</td>
</tr>
<tr>
<td>France</td>
<td>0.000</td>
<td>0.000</td>
<td>0.386</td>
</tr>
<tr>
<td>Greece</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Italy</td>
<td>0.030</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Japan</td>
<td>0.899</td>
<td>0.004</td>
<td>0.021</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.054</td>
<td>0.089</td>
<td>0.175</td>
</tr>
<tr>
<td>Spain</td>
<td>0.099</td>
<td>0.013</td>
<td>0.008</td>
</tr>
<tr>
<td>UK</td>
<td>0.006</td>
<td>0.668</td>
<td>0.079</td>
</tr>
<tr>
<td>US</td>
<td>0.592</td>
<td>0.000</td>
<td>0.385</td>
</tr>
</tbody>
</table>

Next we consider stability in the three interest rate regimes. Stability requires comparison of the responsiveness in the particular interest rate regime with the interest rate in that regime.  

26 Specifically, we require \( \gamma_i^T > r_i (1 - \beta) \) in each regime.

27 Failure to satisfy stability criteria when the interest rate is negative should not imply that agents do not lend because governments do not face a binding intertemporal budget constraint. The more they borrow, the more revenue they receive due to the negative interest rate.
Table 8 Long Run Debt as % of GDP

<table>
<thead>
<tr>
<th>Country</th>
<th>First Regime</th>
<th>Second Regime</th>
<th>Third Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>86.49</td>
<td>141.53</td>
<td>71.99</td>
</tr>
<tr>
<td>Canada</td>
<td>50.61</td>
<td>98.40</td>
<td>72.74</td>
</tr>
<tr>
<td>France (unstable)</td>
<td>83.36</td>
<td>81.50</td>
<td></td>
</tr>
<tr>
<td>Greece (unstable)</td>
<td>89.20</td>
<td>78.72</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>57.57</td>
<td>105.35</td>
<td>-</td>
</tr>
<tr>
<td>Japan</td>
<td>25.38</td>
<td>79.75</td>
<td>188.75</td>
</tr>
<tr>
<td>Portugal (unstable)</td>
<td>67.27</td>
<td>84.79</td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>18.53</td>
<td>73.15</td>
<td>21.03</td>
</tr>
<tr>
<td>UK</td>
<td>(unstable)</td>
<td>48.59</td>
<td>40.58</td>
</tr>
<tr>
<td>US</td>
<td>50.23</td>
<td>54.95</td>
<td>69.07</td>
</tr>
</tbody>
</table>

It is interesting to compare our estimates for long-run debt with the Maastricht requirements in the European Monetary Union that debt be no more than 60% of GDP. Eichengreen and Panizza (2014) argue that the high-debt European countries are more risky than typically believed because they would have to run very high surpluses for very long periods of time, behaviors which are unprecedented for most of them, to bring debt back to the required 60% of GDP. However, our estimates imply that only two countries, the UK and Spain, have long-run debt as low as the 60% benchmark. The remaining eight countries require less severe increases in surpluses.

Figures 6 and 7 illustrate this claim in the context of our model using phase diagrams. We set the long-run debt value of 60% of GDP by adjusting the constant in the fiscal rule (equation 14) and retain the other estimated policy parameters \((\beta, \gamma_3)\). We use the mean interest rate during the third regime in equation (2) to project the path of debt forward for each country. Our projections begin in 2014. The dotted lines represent projections which reach the 60% target, while the solid lines represent our estimated projections which reach the estimated long-run values in the third interest rate regime. The eight countries with long-run debt higher than 60%, require smaller future surpluses. These differences are particularly pronounced for France, Greece, Italy, Japan, and the US. These countries would require surpluses larger than their historical maximums for periods between 7 and 15 years to reach the 60% target. However, given that estimated fiscal rules reveal no plans by these countries to reach the 60% target, the projected surpluses are much smaller. Even so, the trajectory toward our estimated long-run value of debt can involve significant risk, which we investigate in the next section.
Figure 6: Countries with long run values of debt greater than 60% of GDP
4 Fiscal Risk in Ten High-Debt Developed Countries

A country is insolvent and creditors will not lend, when the debt-surplus pair is above the effective fiscal limit. When the debt-surplus pair is below, but close to the effective limit (such as point B in Figure 1), the country is solvent, but at risk of receiving shock(s) which push it over and therefore is at risk of a fiscal crisis. If we could estimate the effective fiscal limit, then we could estimate the risk of insolvency as in Daniel and Shiamptanis (2012).

For countries which lost access to markets during the financial crisis, we can estimate an effective fiscal limit and use this to infer a fiscal limit. The estimated fiscal rule, together with the debt accumulation equation, can be used to project a path for debt at the risk-free interest rate, beginning at the value of the debt-surplus pair for which the country lost access. The peak along that path is greater than or equal to the fiscal limit on debt for that country. The problem with this assessment is that it yields a value for a fiscal limit only after a country has crossed its effective fiscal limit. We would like to use fiscal limits to predict crises, not just to describe those which have occurred. Specifically, we would like to assess which high-debt countries are most at risk of a solvency crisis before they lose access to markets.

For countries which have not lost access to financial markets, there are two approaches to estimating an effective fiscal limit. We could use a debt limit, estimated in a surplus equation for which responsiveness falls at some level of debt, sufficiently for the system to become unstable. Alternatively, we could combine knowledge of a fiscal limit with
estimation of a fiscal rule in which responsiveness is independent of debt. We have not been successful at identifying a debt limit in regressions for which responsiveness depends on the level of debt. Therefore, we use the second method.

We make an assessment of fiscal risk based on the available information for the fiscal limit. We do not use estimates for the top of the Laffer curve as in Bi (2012), Bi, Leeper & Leith (2013) for three primary reasons. First, it is not obvious that the political system could actually raise all tax rates to reach the peak of the Laffer curve. Second, some tax rates, particularly consumption tax rates, do not necessarily yield a Laffer curve. And, third, the ability to reduce government spending is also critically important in determining the maximum surplus, and the Laffer curve estimate has no information about the potential for government spending reduction.

Instead, we use estimates of the long-run value of debt and historical values for debt to construct a lower bound on the fiscal limit. We compare the lower bound on the fiscal limit with our projections for debt. If projections for debt breach the lower bound on the fiscal limit, then we put these countries into a "high risk" category. Other countries go into a "low risk" category. It is important to realize that countries in our high-risk category are not deemed to be insolvent. They have violated the lower bound on the fiscal limit, but not the fiscal limit itself. And we do not know how close the lower bound is to the true fiscal limit.

We divide countries into these two risk classes in the years following the global financial crisis. For countries which lost access to financial markets after the financial crisis, we can determine whether our criterion put them into the high-risk category prior to their crisis. The measure warns of possible future fiscal crises for countries which have entered the high-risk category more recently.

4.1 Lower Bound on the Fiscal Limit

Our estimates provide information on a lower bound for the fiscal limit by providing an estimate for the long-run value of debt in each interest rate regime. Given that agents were lending during each regime, their expectation must have been that the fiscal limit was above the long-run value of debt. This is because agents would not lend into a regime for which debt projections lead to a value of debt above the fiscal limit.

We have additional information on the lower bound for the fiscal limit, given by the historical experience with debt. All countries had access to financial markets over our 1970-2007 sample, implying that the peak of debt projections, beginning from historical values for debt and the surplus, must have been below the fiscal limit. We use the historical
values of debt to infer ability to pay for the same reasons that private credit markets use a household’s history of borrowing and lending to set credit limits. Our assumption is that a country which has successfully serviced a particular level of debt could do so again with third-regime interest rates which are no higher than rates prevailing during the historical maximum.\textsuperscript{28}

\textbf{Table 9 Lower bound on fiscal limit}

<table>
<thead>
<tr>
<th></th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>141.53</td>
<td>141.53</td>
<td>141.53</td>
<td>141.53</td>
<td>141.53</td>
<td>141.53</td>
<td>141.53</td>
</tr>
<tr>
<td>France</td>
<td>83.36</td>
<td>83.36</td>
<td>83.48</td>
<td>85.89</td>
<td>89.92</td>
<td>92.33</td>
<td></td>
</tr>
<tr>
<td>Greece</td>
<td>106.85</td>
<td>109.31</td>
<td>126.78</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Italy</td>
<td>117.17</td>
<td>117.17</td>
<td>117.17</td>
<td>123.14</td>
<td>128.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>188.75</td>
<td>189.20</td>
<td>202.08</td>
<td>204.35</td>
<td>217.31</td>
<td>223.06</td>
<td>229.94</td>
</tr>
<tr>
<td>Portugal</td>
<td>84.79</td>
<td>84.79</td>
<td>85.66</td>
<td>97.69</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>73.15</td>
<td>73.15</td>
<td>73.15</td>
<td>84.44</td>
<td>92.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>75.79</td>
<td>75.79</td>
<td>75.79</td>
<td>81.83</td>
<td>85.82</td>
<td>87.31</td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>71.92</td>
<td>80.77</td>
<td>97.57</td>
<td>105.05</td>
<td>109.51</td>
<td>110.97</td>
<td></td>
</tr>
</tbody>
</table>

For the lower bound on the fiscal limit for 2008, we use the maximum of our two estimates, long-run debt and the peak of the debt projection from the historical maximum value of debt and the associated surplus. Going forward, we update our estimate of the lower bound based on new historical maxima as countries successfully service higher debt and remain in the market. Table 9 contains values for the upper bound for each year together with the source of the 2008 bound, either the long-run value of debt or the historical maximum. Values change over time only due to updating the historical maximum.

4.2 High-Risk and Low-Risk Countries

Next, we use our estimated fiscal rule (equation 14), together with the equation (2) for the evolution of debt, to project the paths of debt forward for each country using the mean...
value of the interest rate during the third interest-rate regime. We make projections for each country for each year beginning with 2008. Figure 8 contains phase diagrams for each country together with the projection paths initiated from the debt-surplus pair in each year, and our estimate for the lower bound on the fiscal limit for 2008.

![Figure 8: Projected paths for the U.S. and Japan](image)

Now, consider countries which are candidates for our high-risk category. US debt rose so much in 2008 that it exceeded our estimate of the 2008 lower bound on the fiscal limit of 71.92% of GDP. This placed the US into our high-risk category. However, the US retained access to credit markets, implying that agents believed the US could follow its fiscal rule back to the long-run value of debt. The 2008 experience raised our 2009 estimate of the lower bound on the fiscal limit to the peak of the projection from the 2008 debt-surplus values. In 2009, the debt-surplus pair was again above the 2009 lower bound of 80.77% of GDP, placing the US in the high-risk category. Since the US retained credit-market access, our 2010 estimate of the lower bound on debt rose again to the peak of the 2009 projection of 97.57% of GDP. The US remained in the high-risk category, with our estimate of the lower bound rising until 2013. At this point, the debt-surplus pair fell below our estimate of the lower bound, and the US exited the high-risk category.

In contrast, consider Japan. It entered the high-risk category in 2008. Every year thereafter, the debt surplus-pair has been above a rising lower bound for the fiscal limit.

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29We use the interest rate in the third regime because interest rates during the financial crisis can have default premia – our projections are based on a risk-free rate. For countries with higher interest rates due to default premia, debt accumulates faster than along our projections, partially explaining the upward shifts of the projections.
Therefore, the country became at high risk in 2008 and remains at high risk after 2014.

Figure 9: Projected paths for the U.K., France, Spain and Italy

In Figure 9, we present projection paths for other countries which followed the Japanese pattern of entering the high-risk category and not exiting. The UK and France entered in 2010, and Spain and Italy entered in 2012. All remain in the high-risk category at the end of 2014 with debt-surplus pairs always above the previous year’s projection, and therefore above the rising lower bound on the fiscal limit. Although Spain never lost access to the markets, it did request an official EU loan in 2012 due to the high market interest rates it was facing. For the UK, the debt-surplus pairs for 2013 and 2014 appear to be getting closer to the previous year’s projections, suggesting that they are "almost" able to return to their stable fiscal rule which would bring debt down and return them to low-risk status.

For the US and the UK, broadening our definition of historical maximum to the years
following World War II eliminates them from the high-risk category. Reinhart’s web site lists 1946 debt for the US and UK as 121.3% and 237.3% of GDP, respectively. Both raise our estimates of the lower bound sufficiently to place them at low risk. For the remaining four countries, France, Italy, Japan, and Spain including the years following World War II does not mitigate our assessment that they belong in the high-risk category.

![Figure 10: Projected paths for Greece and Portugal](image)

For Greece and Portugal, our risk assessment predicts the crises that occurred. Figure 10 shows that the shock in 2008 put Greece immediately into the high-risk category by raising debt above the lower bound on the fiscal limit. Greece was able to borrow at the end of 2009 with debt at 126.78% of GDP but lost access in early 2010, placing the actual fiscal limit on Greece at 126.78% of GDP.\(^{30}\) Our assessment that Greece entered the "high risk" category as early as 2008 or that its actual fiscal limit is 126.78% of GDP could be incorrect due to inaccurate data. Greek data over this period was substantially revised, implying that creditors could have been forming expectations based on misleading data. As of November 2009, before Greece lost market access, the OECD posted a value for debt below the fiscal limit and was forecasting a lower deficit and debt for Greece for the end of 2009.\(^{31}\) Therefore, based on the data available at the time, Greece did not become at "high risk" of a fiscal crisis until 2009, and the crisis occurred shortly thereafter in early 2010.\(^{32}\) Based on the older data, markets could have put Greece’s fiscal limit much lower,

\(^{30}\)The 2009 value is the peak of the projection beginning in 2009.

\(^{31}\)In November 2009, the forecast for the 2009 Greek debt and primary surplus were 111.5% and -8.17% of GDP, respectively.

\(^{32}\)Data revisions cannot account for the very early crisis prediction from the threshold model because
since they were refusing to lend when data available at the time yielded a maximum debt projection of 111.5% of GDP.

In Portugal, the debt projection from 2009 values breached the country’s lower bound, and put the country into the high-risk category in 2009. Portugal lost access to credit markets two years later in early 2011, placing the actual fiscal limit below 97.69% of GDP, the peak of the debt projection from the 2010 values.

The only countries which have remained in the low-risk category over all years following the advent of the financial crisis are Belgium and Canada, as shown in Figure 11. It is notable that these countries have not traditionally been low-debt countries. Their low-risk status reflects their ability to follow a stable fiscal rule even when debt is high. Even though debt rises following the initial shock, the path for the debt and surplus remains close to original projections with debt projections reaching a long-run equilibrium without nearing the lower bound on the fiscal limit. They quickly checked the explosive behavior of debt following the shock, in contrast to other countries for which debt embarked on a more explosive path. It is notable that the difference in status for these countries is not due to a low debt level – they have high debt – but due to their ability to keep debt and the surplus close to their original projection paths after the financial shock.

Figure 11: Projected paths for Belgium and Canada.

Our assessment puts eight of the ten countries into the high-risk category sometime after the world-wide financial crisis beginning in 2008. Two of those countries, Greece and Portugal, did lose access to financial markets after our method designated them as the threshold model indicates debt above the fiscal limit as early as 1993 in one case and 2000 in another.
high risk. Spain, requested an official loan due to very high market interest rates, but did not actually lose market access. The US has exited the high-risk category, and with an alternative estimate of the lower bound on the fiscal limit, the US and the UK never entered the high-risk category. The remaining four high-risk countries, France, Italy, Japan and Spain, potentially face risk of a fiscal crisis. We cannot actually measure how high the elevated risk is because we do not know how far our lower bound on the fiscal limit for debt is from the true fiscal limit. However, the fact that debt-surplus pairs continue to exceed a rising estimate of the lower bound is worrisome.

5 Conclusion

We propose and implement a simple data-based method for separating high-debt, developed countries into two categories determined by risk of a fiscal solvency crisis. Under the assumption that the government can commit to repay based on ability, we show that a government loses access to credit markets when debt crosses an effective fiscal limit, beyond which the government is insolvent. The effective fiscal limit is the highest stable path for debt whose peak is less than or equal to either a debt limit, beyond which the debt-surplus system becomes unstable, or a fiscal limit, defined as the highest value of debt the country can repay.

Implementation requires estimation of a fiscal rule, whereby the surplus adjusts to debt. If surplus-responsiveness to debt is high enough, then the dynamic system is globally stable. Empirically, the surplus-responsiveness to debt is time-varying. If surplus-responsiveness varies with debt and falls with high debt, sufficiently to create instability, then we can use the estimated surplus equation to identify a debt limit, the value of debt beyond which the system becomes explosive.

We follow Bohn (1998, 2008) and Ghosh et al. (2013) and estimate a threshold model for each country in which surplus-responsiveness depends on the level of debt. We find that the surplus-responsiveness to debt is highly varied across countries, with fiscal fatigue occurring for only three of our ten countries: Greece, the US and the UK. However, for these countries, the implied limits are too low, with Greece and the US borrowing when actual debt-surplus pairs are above the estimated effective fiscal limit over the 1970-2007 period.

We propose an alternative source of time-variation in responsiveness and allow the responsiveness to depend on the interest rate. The estimates yield surplus-responsiveness increasing in the interest rate for all ten countries. The consistent behavior of surplus-
responsiveness with respect to changes in the interest rate contrasts with the varied behavior with respect to changes in debt. We suspect that the reduced responsiveness, attributed to fiscal fatigue in the threshold model for three countries, is actually due to the fall in interest rates instead of to the rise in debt. However, the interest rate model cannot identify a fiscal limit. Therefore, the data does not identify a fiscal limit for a country which has not crossed one.

We assess solvency risk for countries following the financial crisis using our estimated fiscal feedback rule in which responsiveness depends on the interest rate together with an estimate of the lower bound on the fiscal limit. Since agents were lending over the sample, the equilibrium value of debt, toward which each economy was converging, must have been below its fiscal limit. Our estimates identify the long-run value of debt. Additionally, projections of debt from any historical value, including the historical maximum, must have been below the fiscal limit. We set the lower bound on the fiscal limit equal to the maximum of the long-run value and the peak of the projection from the historical maximum. We separate countries into high and low risk categories in each year following the financial crisis, depending on whether the projection of debt beginning in that year, using the estimated fiscal rule and interest rate, remains below the estimated lower bound for the fiscal limit.

We find that only Belgium and Canada retained low-risk status in all years following the financial crisis. The criterion which sets these countries apart is not low debt, but their ability to rapidly check explosive debt behavior following a major surplus shock. Our method predicts the crises which occurred in Greece and Portugal about two years in advance. When the 2008 lower bound on debt is determined using the 1970-2007 sample, both the US and the UK entered high risk status with the US returning to low risk in 2013. However, if we extend the definition of historical maximum to include post World War II history, then neither country enters the high risk category. France, Italy, Japan, and Spain have all entered the high risk category and show no signs of returning to their estimated fiscal rules, such that debt would begin to fall toward its long-run equilibrium value. Therefore, our method warns of future solvency crises in these four countries.

References


