A Pure Hedonic Theory of Utility and Status: Unhappy but Efficient Invidious Comparisons*

Pascal Courty, University of Victoria, Canada
Merwan Engineer, University of Victoria, Canada

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Abstract: We model each individual’s utility as the sum of her primary utility, which depends on her consumption, and her status utility, which depends on her comparing her utility to others’ utilities. The utility functions are, therefore, implicit functions of each other. As long as status comparisons are not too intense, the introduction of status utility does not affect either the competitive equilibrium or the set of efficient allocations. However, status utility may substantially reduce average utility and dramatically increase utility inequality. Also, competitive allocations are no longer in the core of the economy when voluntary transfers are made to the “enemy of my enemy”. The analysis identifies new welfare issues and political economy conflicts. Equating utility with happiness operationalizes the theory and provides an explanation to the puzzle of why invidious comparisons can generate so much unhappiness without much inefficiency.

JEL: D10, D60.

Keywords: Conspicuous consumption, inequality, happiness, rat race, reference group, status, utility, welfare.

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Our envy always lasts longer than the happiness of those we envy.

duc de la Rochefoucauld (1665)

Apart from economic payoffs, social status seems to be the most important incentive and motivating force of social behavior.

Harsanyi (1976, p. 204)

1 Introduction

There is now a substantial body of economic theory that models interpersonal status comparisons in ways that generate inefficient allocations. This body of theory is motivated by the classic works of Veblen (1899) and Duesenberry (1949) as well as empirical studies that find that individuals report lower happiness when the income of their reference group increases. Arrow and Dasgupta (2009) accept that status motivated comparisons may generate much unhappiness. However, they are skeptical that such comparisons generate large allocative inefficiency because of the lack of direct empirical evidence. We argue that this suggests an invidious comparison - efficiency paradox. The term “invidious comparison” is used by Veblen (1899) to describe pervasive status comparisons that induce chronic dissatisfaction for the relatively poor and restless straining for the relatively rich. In this paper, we address the invidious comparison - efficiency paradox with a simple theory that reconciles allocative efficiency with widespread and deep utility losses from status comparisons.

This paper develops a pure hedonic theory of utility and status. We investigate whether allocation and utility distortions occur when the status function involves comparisons of cardinal utilities across individuals, not independent arguments in the utility function as used in the literature. We name this class of status function “status utility” because utilities are being compared. Status utility captures how well an individual is doing in utility terms relative to her reference group. An individual’s utility affects her status utility and vice versa. For status conscious individuals, where maximizing their own utility is their end goal, utility comparison is the ultimate basis for comparing individuals. Equating cardinal utility with measures of happiness from the subjective well-being being literature operationalizes the theory. Happier individuals have higher status and are happier for it. We motivate this approach with respect

\[ ^{1}\text{Clark et al. (2008) review both this theory literature as well as the empirical well being literature that supports allocative inefficiencies. We review the literature in Sections 3.1 and 4.1. Frank (1985, 2010), Layard (1980, 2010) and Oswald (1983) have long urged us to accept the sad reality that relative consumption is a major negative psychological externality that, like pollution, could be taxed to public benefit.} \]
to the literature in Section 2. This introduction describes the theory as providing a useful and tractable framework.

In the model, each agent’s utility is separable in their primary utility and their status utility. Primary utility is the standard cardinal utility function defined on consumption and leisure. Status utility is a linear function of the difference between own utility and the weighted sum of utilities of other agents in the reference group. The utility functions are therefore implicit functions of each other. With this set up, and assuming status comparisons are not too intense, we derive the following key results: (i) The competitive equilibrium is unaffected by the introduction of status utility. (ii) The set of Pareto efficient allocations is unaffected by status utility. (iii) Survey evidence finding positional concerns suggests the importance of status utility, without resort to positional preferences over goods. (iv) Status utility generates network effects that typically increase utility inequality. (v) In the most relevant cases, status utility is negative-sum and equity policy increases average utility. (vi) Utilitarian allocations reduce negative status externalities by allocating more to those who compare themselves more intensely to others. (vii) Competitive allocations are not in the core of the economy when the rich benefit by making gifts to the “enemy of my enemy”. (viii) The very rich may pursue status in ways that resemble descriptions of conspicuous consumption in Veblen (1899).

Including utility comparisons into the standard model need not alter key positive and normative results. Neutrality results (i) and (ii) boil down to each agent’s utility ultimately depending on their own primary utility and not being able to manipulate others’ primary utilities. In contrast, the literature departs from the standard model by examining status good comparisons where a subset of goods, termed “positional goods” by Hirsch (1976), generate negative consumption externalities. The more positional is the good, the greater is the externality and distortion. To this conclusion, Arrow and Dasgupta (2009) provide a major qualification. They show that if all goods are equally positional then the externalities cancel out. In their economy, each good is separable in the utility function and there is a negative externality from the absolute level of average consumption of each good. The market equilibrium has no allocation distortions when good externalities are proportionally symmetric across consumption goods. In our model, the balance is naturally struck as agents in effect maximize primary utility. Allocations and primary utilities are the same as in the standard model. This neutrality result survives recursive utility comparisons and all specifications of reference groups.

Our model offers other contrasting perspectives to the literature. Solnick and Hemenway (1998, 2005) and others find dramatic survey evidence for positional concerns. For example, about half of those surveyed prefer a world where they get a lower absolute income when it

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improves their relative income position. However, for vacations only a few respondents choose less vacation time. The result that income is far more positional than leisure in the utility function is at odds with the requirement of Arrow and Dasgupta (2009) that goods be equally positional for externalities to cancel out. In contrast, we show that the survey evidence is consistent with standard primary utility functions when status utility is important (result iii). From this perspective, utility is positional and goods need not be positional. Another contrast with the literature is that we specify reference groups quite generally. The theoretical status literature has narrowly concentrated on two references standards, either the average consumption of the positional good or rank in the income distribution. A notable exception is Ghiglino and Goyal (2010), who specify local positional good comparisons and trace how allocation and price distortions spread non-locally. We explore a wide range of reference groups and show that the social network increases the dispersion of utilities (result iv).

Status utility can affect average utility and utility inequality. When agents have identical status preferences, utility inequality rapidly increases in the weight placed on status utility. However, in this benchmark case, status utility is zero sum and average utility is unaffected by status, a property we call “utility conservation”. This property reveals that primary utility plays the same underlying absolute role as it does in the standard model without status. With asymmetric status preferences, utility conservation fails and status utility may be either positive sum or negative sum. If it is negative sum, then average utility falls.

Status utility is negative sum in empirically relevant cases. The literature finds that poorer individuals tend to compare more intensely to others. In the model, this implies that less well off agents place greater weight on status utility. Also, individuals tend to compare upward, towards those richer than themselves. This implies that agents place greater weight in their reference group on those who are better off than themselves. We show that either of these patterns of status comparisons can generate negative-sum status utility (result v). For example, this will be the case when income growth accrues only to the richest. The utility gain to them can induce an offsetting utility loss on others that is negative-sum overall. Consequently, the growth in average utility may be negligible or even negative in extreme cases. Also, utility inequality increases. These results arise solely from the intensity of upward status comparisons and not from diminishing marginal utility and relative income comparisons (Johansson-Stenman et al. 2002). The literature has neglected this explanation in trying to rationalize the “Easterlin paradox” (Clark et al. 2008, Easterlin 2013), evidence that economic income growth may not have increased average long-term happiness or reduced happiness inequality in some developed nations. As our model does not generate allocative distortions, it also provides an explanation for the invidious comparison – efficiency paradox.
Welfare improving policies are affected by status utility when utility conservation fails. When status utility is negative sum, average utility is increased by policies, like progressive taxation, which reduce utility inequality. An Utilitarian planner, who respects status preferences, allocates more resources to individuals who feel status comparisons more intensely so that they end up comparing downward to others who are made less better off (result vi). The optimal policy generates positive-sum status utility by creating new inequalities.

A positive implication of our model is that the competitive equilibrium is not always in the core of the economy. Agents may be better off deviating from the competitive equilibrium allocation by gift (result vii). In particular, voluntary “spiteful transfers” may occur if giving to the less fortunate benefits the “enemy of my enemy”. We consider two mutually exclusive groups in which intergroup comparisons are far more intense than intragroup comparisons. It is in the interest of the rich agents of a group to coordinate on transfers to their poorest group members. But, this charity is not for the love of one’s brethren but rather to diminish others outside the group.

Lastly, relaxing the assumption of nonsatiation in primary utility implies behavior resembling Veblen’s description of the leisure class. Having become satiated in primary utility, the rich compete for status through conspicuous consumption in the sense of consuming goods obviously beyond the point of satiation rather than give to the poor (result viii). Here, the efficiency equivalence between the status model and the standard model fails, because willful waste is efficient in a world with insatiable status. Both conspicuous consumption and spiteful transfers do not occur in more egalitarian societies.

The paper proceeds as follows. Section 2 describes the model, examples, and the merit of specifying status as relative utility. Section 3 solves the utility system. The implications of invidious comparisons and positional concerns are contrasted with the literature. Section 4 develops the equilibrium and efficiency neutrality results and discusses the invidious comparison - efficiency paradox. The neutrality results do not preclude a political economy of spiteful transfers and conspicuous waste. Section 5 shows that status utility can substantially reduce welfare. An explanation for the Easterlin paradox is proposed, and egalitarian and Utilitarian policies are contrasted. Section 6 concludes by arguing that our theory provides a consistent hedonic methodological approach which is a useful foundation for further inquiry.

2 Agents, Status and Utility

Agents are indexed $i = 1, ..., N$ for $N \geq 2$. Agent $i$’s utility $u_i$ depends on two components, “primary utility” $U_i$ and “status utility” $S_i$. We focus on the case where utility is separable and
additive in its two components:

\[ u_i = U_i + S_i. \] (1)

Primary utility \( U_i \) can be individual specific but does not depend on comparisons with others. For simplicity, we assume that primary utility is an increasing strictly concave function of own consumption \( c_i \) and leisure \( l_i \) and denote it \( U_i \equiv U_i(c_i, l_i) \) for \( i = 1..N \). This specification highlights the trade-off between consumption and leisure and allows ready comparisons with the rat race literature. It is convenient to assume a finite minimum for utility and normalize it \( U_i^{\min} \equiv U_i(0, 0) = 0 \). Primary utility could be specified far more generally than is assumed here.

Our innovation is that status utility \( S_i \) is determined by agent \( i \) comparing her own utility, \( u_i \), relative to the utilities of others, \( u_j \) for \( j \neq i \), as follows:

\[ S_i = s_i \left( u_i - \sum_{j \neq i} \omega_{i,j} u_j \right) \] (2)

where \( s_i \in [0, 1) \) and \( s_i > 0 \) for at least one agent, \( \omega_{i,j} \in [0, 1] \), and \( \sum_{j \neq i} \omega_{i,j} = 1 \). The parameter \( s_i \) is agent \( i \)'s absolute status intensity; it is the weight which agent \( i \) puts on the difference between her own utility, \( u_i \), and the weighted-average utility of her reference group, \( \sum_{j \neq i} \omega_{i,j} u_j \). Weight \( \omega_{i,j} \) is agent \( i \)'s relative status intensity for agent \( j \). If agent \( j \) is not part of agent \( i \)'s reference group, then \( \omega_{i,j} = 0 \). Status utility is a hedonic construct because it involves utility comparisons and values them in the same units as primary utility.

Equation (2) allows for variation across agents in the relative intensity of utility comparisons. If agent \( i \) compares predominantly, to a reference group with higher weighted average utility, then she receives a negative status payoff \( (S_i < 0) \). This suffering, by comparing predominantly with those who are better off, is associated in the status literature with feelings of envy, inferiority and/or low self esteem. On the other hand, if agent \( i \) compares predominantly downward, to a reference group with lower utility, then she receives a positive payoff \( (S_i > 0) \). Benefiting is associated with feelings of superiority, high self esteem and/or “counting your blessings”. Finally note that if utility is the same for all agents, \( u_i = u_j \) for all \( i \neq j \), then \( u_i = U_i \) for all \( i \). Here status utility is \( S_i = 0 \) for each agent and zero sum, \( \sum_i S_i = 0 \), in aggregate.

### 2.1 Examples

Table 1 introduces five leading examples. In the Two Agents case, agents have identical relative status intensities, \( \omega_{1,2} = \omega_{2,1} = 1 \). Identical Status assigns identical absolute and relative status intensity parameters to all agents: \( s_i = s > 0 \) and \( \omega_{i,j} = \frac{1}{N-1} \) for all \( i \) and \( j \). Here the status
Table 1: Examples of Status Functions

<table>
<thead>
<tr>
<th>Status Function</th>
<th>Status Intensity $s_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Agents</td>
<td>$s_i (u_i - u_{-i})$</td>
</tr>
<tr>
<td>Identical Status</td>
<td>$s \left( u_i - \frac{1}{N-1} \sum_{j \neq i} u_j \right)$</td>
</tr>
<tr>
<td>Two Mutually Envious Groups</td>
<td>$s \left( u_i - \frac{1}{n} \sum_{j=n+1}^N u_j \right)$ for $i = 1..n &lt; N$</td>
</tr>
<tr>
<td>One-up Status</td>
<td>$s (u_i - u_{i+1})$ for $i &lt; N$, $S_N = 0$</td>
</tr>
<tr>
<td>All-up Status</td>
<td>$s \left( u_i - \frac{1}{N-i} \sum_{j&gt;i} u_j \right)$ for $i &lt; N$, $S_N = 0$</td>
</tr>
</tbody>
</table>

Note: $u_i(U_i, S_i) = U_i + S_i$. These examples are analyzed in Appendix C.

function is decreasing in the average utility of others. This is analogous to the widely used rat race formulation (Frank (1985), Arrow and Dasgupta (2009)) and to most of the empirical literature where agents compare their own consumption to average consumption. Two Mutually Envious Groups divides the society into groups of sizes $n$ and $N - n$, where $n < N$. Members of a group are not envious of each other, but are equally envious of each member of the other group (their reference group). The last two examples assume that primary utilities are ordered $U_1 < U_2 < \ldots < U_N$ from the least to the most “affluent”. With One-up Status each agent is envious of the person with the next highest utility. With All-up Status each agent is envious of all persons with higher utility. In both cases agent $N$ does not compare to anyone.

The Two Agents example reveals some general properties. The utility of agent 1 is determined by her primary utility and the primary utility of agent 2 as follows (see Appendix C):

$$u_1 = \frac{1-s_2}{1-s_1-s_2} U_1 - \frac{s_1}{1-s_1-s_2} U_2$$

The utility of agent 2 is symmetric with subscripts 1 and 2 switched. If the absolute status intensities are not too large, $s_i \in [0, \frac{1}{2})$ for $i = 1, 2$, then the denominator is positive. Utility is increasing in own primary utility and decreasing in primary utility of the other agent. Conversely, if $s_i \epsilon (\frac{1}{2}, 1)$, then we have the bizarre case where utilities are decreasing in own primary utility and increasing the primary utility of the other agent. In Section 3, we show that $s_i < \frac{1}{2}$ for all $i$ is a tight sufficient condition for ruling out this bizarre case in the general model.

Suppose agent 1 is less affluent, $U_1 < U_2$. When agent 1 is also more status conscious $s_1 \epsilon (s_2, \frac{1}{2})$, then utility inequality increases, $u_1 < U_1 < U_2 \leq u_2$, while preserving the rank ordering of utility the same as that for primary utility. Also, the sum of the utilities is smaller, $u_1 + u_2 < U_1 + U_2$, so that status utility is negative sum, $S_1 + S_2 < 0$. When agents are equally status conscious, $s_1 = s_2 = s$, status utility is zero sum, $S_1 + S_2 = 0$, as “utility conservation” now holds, $u_1 + u_2 = U_1 + U_2$. Status still increases inequality while preserving the ranking of utilities. In fact, $s \rightarrow \frac{1}{2}$ implies dramatic inequality $u_1 \rightarrow -\infty$ and $u_2 \rightarrow +\infty$. Whereas status
utility is zero sum, status considerations predominate for individuals. These results generalize to \( N > 2 \). In Section 5, we show that utility conservation fails with asymmetric status preferences across agents. We also show that the ranking of utilities is usually preserved.

### 2.2 Hedonic Comparisons in Economics and Beyond

Methodologically, our approach is related to the “non-paternalistic altruism” literature, where individual utility (termed social utility) is a function of own and others’ direct utility functions.\(^3\) Bergstrom (1999) formulates utility more generally using own primary utility and the utilities of others. Like us, utility is cardinal and additively separable in its arguments (and forms a row diagonally dominant matrix system). As in the altruism literature, we take the leap that perceived utility is described by a cardinal function that is an accurate conjecture of preferences.

Of course, cardinal utility and utility comparisons have been discussed in economics since Bentham along with notions of happiness. Recently, a subjective well being literature has emerged where many argue that cardinal indicators of happiness are good empirical proxies for cardinal utility. Clark et al. (2008) and Frey and Stutzer (2010) survey this literature as well as the evidence that self-reported happiness scores are often accurately predicted by others.\(^4\) On the theoretical side, Harsanyi (1987) argues that cardinal utility is both a necessary and defensible assumption for interpersonal utility comparisons. In his analysis, everybody has consistent empathetic preferences which satisfy Von Neumann and Mortenstern rationality requirements. Binmore (2007, Section 19.5) agrees with Harsanyi that people can somewhat successfully conjecture the preferences of others, and he extends the analysis to (the anti-sympathetic motivation) envy as requiring knowledge of not just others’ possessions but also their preferences.

The key importance of envy in human society is described in Elster’s (1989) formidable book “The Cement of Society”. He surveys human motivations and ironically concludes that envy, opportunism and honour are the preeminent motivations that “without which chaos and anarchy would prevail” (p.251). Consistent with our formulation of utility, he describes how the target of envy could include another person’s utility function (ability to enjoy goods) and even their good fortune in not having to experience intense envy. Elster (1991, p. 52) describes

\(^3\) Kolm (2006) traces this approach back to Vilfredo Pareto who distinguished what we call primary utility (his “ophelimity” function) from utility which depended on primary utility functions of others. Archibald and Donaldson (1976) discuss nonpaternalistic altruism and derive the welfare implications. Bourlès et al. (2013) describe social utility as a network and solve the Nash equilibrium of a gift giving game. Interpersonal utility comparisons that enter positively (altruism) and negatively (status or envy) are not necessarily contradictory if they influence different preference domains and govern non-overlapping choices.

\(^4\) The literature distinguishes different types of utility. Regarding decision making, Benjamin et al. (2012) report: “On average, SWB and choice coincide 83 percent of the time in our data” (p. 2085). Zou et al. (2013) find that self-reported happiness and informant ratings are equally valid.
happiness as the object of envy and, anticipating our results, notes that this can generate a
vicious spiral that diminishes happiness. The importance of having and knowing a reference
group is also suggested by the recent psychological research where envy intensity is positively
correlated with interpersonal knowledge and counterfactual thinking.\textsuperscript{5} It is debated whether
interpersonal comparisons are the expression of hardwired preferences, that could result from
evolutionary selection (see Eaton and Eswaran 2003, Rayo and Becker 2007, Binmore 2009), or
play an instrumental role (see Ireland 1994, Bagwell and Bernheim 1996, Postlewaite 1998). We
follow the approach taken by others (e.g. Hopkins and Kornienko 2004, Arrow and Dasgupta
2009) in that we do not take a stand on the issue and model invidious externalities as a preference.

Some direct empirical support for invidious utility comparisons is found in research outside
of economics. Shin and Johnson (1978) explain self-reported happiness with a measure where
“respondents compare themselves with the people whom they know in terms of life enjoyment”
(p. 481). This measure proxies our status utility. It enters positively and significantly as
the strongest predictor of happiness in all their regressions, suggesting intense invidious utility
comparisons. Michalos (2012) develops multiple discrepancy theory to generalize research on
happiness and life satisfaction. Using student survey data across 39 countries, he finds strong
evidence consistent with intense invidious comparisons. Happiness (and/or life satisfaction)
comparisons with peers is the second most important domain in determining own happiness
(and/or life satisfaction). In Sections 3.1 and 3.2, we review the economics literature on in-
vidious comparisons and argue that it provides compelling indirect evidence for relative utility
comparisons.

3 Utility, Invidious Comparison, and Positional Concerns

The system defined by equations (1) and (2) for \(i = 1,\ldots,N\) can be written in matrix notation

\[
u = U + s(I - \omega)u\]

where \(u\) and \(U\) are vectors, \(s\) is a diagonal matrix with elements \(s_i\), \(I\) is the identity matrix and \(\omega\)
is a matrix with diagonal elements zero and off diagonal elements \(\omega_{i,j}\). Define \(A \equiv I - s(I - \omega)\),
and assume for now that \(\det A \neq 0\). From \(Au = U\) we obtain the solution \(u = BU\) where
\(B \equiv A^{-1}\). Thus, utilities are explicitly related to primary utilities according to

\[
u_i = b_{i,i}U_i + \sum_{j \neq i} b_{i,j}U_j\]

\textsuperscript{5}See van de Ven et al. (2011) and van de Ven and Zeelenberg (2014). Lebreton et al. (2012) find that coveting
what others want may be hardwired into the brain.
where $b_{i,i}$ and $b_{i,j}$ are the own and cross marginal utilities from changes in primary utilities, $b_{i,i} = \frac{\partial u_i}{\partial u_i}$ and $b_{i,j} = \frac{\partial u_i}{\partial u_j}$. Though the own primary utility effects $b_{i,i}$ are related to the parameters of the social utility functions $(s, \omega)$ in a complex way, we argue that that they should be positive. This always holds when the absolute status intensities are not too large.

**Assumption 1.** $s_i < 1/2$ for all $i$.

**Lemma 1.** (a) If Assumption 1 holds, inverse matrix $B$ is well defined and satisfies $b_{i,i} > 0$. (b) If Assumption 1 is violated such that $s_k + s_i \geq 1$ for any two agents, then there is a matrix of relative weights $\omega$ such that either $B$ is not well defined or $b_{i,i} < 0$ for at least one $i$.

Assumption 1 provides a tight sufficient condition for existence of a solution for any $\omega$. This solution rules out $b_{i,i} \leq 0$ for any agent $i$. With $b_{i,i} < 0$ agent $i$’s prefers to deny herself consumption to deny other agents utility. This bizarre behavior is reminiscent of the expression “cut off your nose to spite your face”. We do not believe that such extreme behavior is at work in most societies, and therefore concentrate on status parameters $(s, \omega)$ that give $b_{i,i} > 0$ for all $i$. Thus, when considering general matrices $\omega$, we maintain Assumption 1 in the rest of the paper.

The existence of a solution in Lemma 1 is derived from the fact that under Assumption 1 matrix $A$ is row diagonally dominant, $|a_{i,i}| > \sum_{j \neq i} |a_{i,j}|$, with positive diagonal entries $a_{i,i} > 0$ (Horn and Johnson 1991). Further, under Assumption 1, the solution of this matrix system satisfies the following properties which describe the mapping from primary utilities to utilities.

**Proposition 1.** The marginal utilities from changes in primary utilities are as follows: (i) Own effect, $b_{i,i} > 1$ for $s_i > 0$ (and $b_{i,i} = 1$ for $s_i = 0$); (ii) Own effect dominates effect on others, $b_{i,i} > |b_{j,j}|$ for $j \neq i$; (iii) Total effect invariance, $\sum_j b_{i,j} = 1$.

For status conscious agents, effects (i) and (iii) imply $\sum_{j \neq i} b_{i,j} < 0$ so that the total status comparison effect is always negative. In many cases, $b_{i,j} < 0$ for all $j \neq i$ (e.g. Two Agents and Identical Status economies). However, $\sum_{j \neq i} b_{i,j} < 0$ does not rule out $b_{i,j} > 0$ for a subset of $j$. For example, $b_{i,j} > 0$ for up to $n = N - 1$ agents in the Two Mutually Envious Groups economy. The intuition for $b_{i,j} > 0$ is that agent $i$ does not compare herself much to $j$ but does compare herself intensely to others who in turn compare themselves intensely to agent $j$. Thus, agent $i$ indirectly benefits when agent $j$ is better off.\(^6\) Finally, as Proposition 1(iii) implies $x \sum_j b_{i,j} = x$ for any constant $x$ (the vector $1^N$ is an eigenvector of $B$ with eigenvalue 1), we immediately have the following result.

\(^6\)For example, with $N = 3$ we have $b_{1,2} = s_1[s_3 \omega_{1,3} \omega_{3,2} - (1 - s_3) \omega_{1,2}] / \det A$, where $\det A > 0$. Then $b_{1,2} > 0$ when agent 1 does not compare herself too much to agent 2 ($\omega_{1,2}$ small) but compares herself to agent 3 ($\omega_{1,3} > 0$) who in turn compares herself to agent 2 ($\omega_{3,2} > 0$ and $s_3 > 0$). In Section 4.3, we explore when agent $i$ gives to agent $j$ as a way of spiting her enemies.
Corollary 1. If primary utilities are equal for all agents, \( U_i = U_j \) for all \( i \neq j \), then utility is equal to primary utility, \( u_i = U_i \), and status utility is zero, \( S_i = 0 \).

Corollary 1 also implies that if the primary utilities of all agents change by the same amount, \( U_i - U'_i = U_j - U'_j \) for all \( i \neq j \), then so does utility, \( u_i - u'_i = u_j - u'_j \), and there is no change in status utility, \( S_i = S'_i \) for all \( i \). Although utility is unaffected by status in these symmetric situations, it is otherwise except for a specific class of status functions as shown in Section 5.

To get a different perspective on patterns of interdependent relationships, we solve the matrix system recursively. Rearranging equation (3) gives \( u = (I - s)^{-1} U - s \omega u \), where \( s \) is a diagonal matrix with elements \( \frac{\omega_i}{1 - \omega_i} \in (0, 1) \). Since \( u = BU \) it follows that \( u = \left( (I - s)^{-1} - s \omega B \right) U \), and we obtain the recursive expression \( B = (I - s)^{-1} - s \omega B \). Substituting this equation into itself gives the series\(^7\)

\[
B = \left[ I - s \omega + [s \omega]^2 - [s \omega]^3 + [s \omega]^4 - \ldots \right] (I - s)^{-1}
\]

The component matrices in the first bracket alternate in sign. The first matrix \( I \), corresponds to the own primary utility effect. It affects \( b_{i,i} \) but not \( b_{i,j} \). The second matrix, \(-s \omega \), involves direct status comparisons where each agent \( i \) compares herself to those in her reference group. This direct negative feedback affects \( b_{i,j} \) but not \( b_{i,i} \). The third matrix, \([s \omega]^2\), captures the indirect positive feedback from all utilities negatively impinging on the utilities of agents \( j \) in \( i \)’s reference group. This indirect positive effect is loosely described by the adage “the enemy of my enemy is my friend”. The next matrix, \(-[s \omega]^3\), is negative as it loosely describes “the enemy of the enemy of my enemy is my foe”. Similarly, we get alternate positive and negative feedbacks for higher order terms. Agent \( i \) benefits from agent \( j \) being better off, \( b_{i,j} > 0 \), only if the indirect positive feedback effects dominate both the direct and indirect negative feedback effects. The rest of this section interprets the empirical evidence on invidious comparisons and positional concerns using our model of hedonic status. It can be read separately.

### 3.1 Invidious Comparisons: Empirical Support

The term (“invidious”) is used in a technical sense as describing a comparison of persons with a view to rating and grading them in respect of their relative worth of value ... with which they may legitimately be contemplated by themselves and by others.

\(^7\)It can be readily verified that \( BA = I \), where \( A = (I - s) [I + s \omega] \). The series omits the transversality term, \( \lim_{k \to \infty} [-s \omega]^k B \), because it is equal to zero under Assumption 1. Consider \( [-s \omega]^K = (-1)^K s \omega^K \). Using the sub-multiplicative property of the max norm (defined as \( \|X\|_{\text{max}} = \max_{i,j} X_{i,j} \) for matrix \( X \)), we obtain \( \|[-s \omega]^K\|_{\text{max}} = \|s\omega\|^K\|\omega^K\|_{\text{max}} \), where \( \|s\omega\|^K\|\omega^K\|_{\text{max}} = \left( \frac{s}{1 - s} \right)^K \) and \( \|\omega^K\|_{\text{max}} \leq 1 \). Under Assumption 1, \( \lim_{K \to \infty} [-s \omega]^K B = 0 \) and the series \( \sum_{k=1}^{\infty} [-s \omega]^k \) converges to a well-defined matrix.
Proposition 1’s implication that people compare themselves to some reference group and do so negatively in some average sense ($\sum_{j \neq i} b_{i,j} < 0$) goes a long way back in social science. In Veblen (1899)’s rich sociological description, invidious comparison is the mechanism that drives an evolutionary dynamic of differentiation and emulation. Duesenberry (1949) also invokes the maintenance of self-esteem as a basic drive which is achieved through what he coined a “demonstration effect” to describe the greater frequency of exposure to higher living standards as leading to greater consumption. In examining international evidence, Easterlin (Easterlin, 2013) finds that happiness increases with income in a cross section of individuals at a point in time in a country, but it paradoxically does not increase on average over time within the country. The Easterlin paradox has triggered a controversial literature on whether the impact of income on happiness is zero-sum (Clark et al. 2012). Subsequently, the role of relative income has been widely examined at the individual level by explaining reported happiness (subjective well being) with own income and the income of a reference group.

Consider the log-linear specification used by Clark et al. (2008) in their survey:

$$u_{i,t} = \beta_1 \ln(y_{i,t}) + \beta_2 \left[ \ln(y_{i,t}) - \ln(y_{i,t}^*) \right] + Z_{i,t}' \nu$$

(5)

where $u_{i,t}$ is a measure of the “happiness” of agent $i$ at time $t$, $y_{i,t}$ is the income of agent $i$, $y_{i,t}^*$ is average income of $i$’s reference group, and $Z_{i,t}$ is a vector of control variables. The reduced form estimate of own income usually enters significantly positively, $\beta_1 + \beta_2 > 0$, in explaining happiness. Most studies in developed countries find that the coefficient on reference income is significantly negative, $-\beta_2 < 0$.\(^8\) Whereas this literature does not establish causality, the presumption is that intense comparisons reduce well-being. Consistent with the Easterlin paradox, a number of studies find that the coefficient on reference income is of the same magnitude as own income, that is, $\frac{\beta_2}{\beta_1 + \beta_2}$ is close to one.\(^9\) Other studies find that relative income partially offsets income, $\frac{\beta_2}{\beta_1 + \beta_2} \in (0, 1)$, which is interpreted as evidence that income is positional.\(^10\)

\(^8\)A positive relative income effect has been found in countries undergoing rapid transitions (Senik 2008). Here income comparisons reveals that others are doing well and is taken as a signal of higher future own income. We do not consider this “tunneling effect” (Hirschman and Rothschild 1973) because it is not empirically dominant (Clark and Senik 2010) and arises from imperfect information rather than preferences.

\(^9\)Clark and Oswald (1996), Luttmer (2005), Ferrer-i Carbonell (2005), Knight et al. (2009), Senik (2009), Helliwell and Huang (2010), Layard et al. (2010).

\(^10\)Arrow and Dasgupta (2009) use $\frac{\beta_2}{\beta_1 + \beta_2} = \frac{1}{3}$, citing Blanchflower and Oswald (2004) analysis of American GSS data. Clark et al. (2008, p. 111) settle on the higher figure of $\frac{2}{3}$. Van de Stadt et al. (1985) reach a similar conclusion using a different methodology associated with the Leyden School. Praag and Ferrer-i Carbonell (2004) summarizes this literature, and Chapter 8 discusses several studies where income comparisons offset up to 80% of the individual effect. On the opposite end of the debate, Deaton and Stone (2013) examine Gallup surveys and
We argue that the evidence is also consistent with our model of status utility where it is utility that is positional. This distinction is important as allocations are efficient in our model. Equation (5) matches, for example, the Identical Status economy once one associates utility to happiness and primary utility to the logarithm of income. One obtains 
\[ u_i = b_{i,i} U_i - (b_{i,i} - 1) U_i^*, \]
where \( U_i^* \) is the average of others’ primary utilities. The ratio of the reference group to own effect, \( \frac{b_{i,i} - 1}{b_{i,i}} = \frac{(N-1)s}{N-1-s} \), can take any value in \((0, 1)\) as \( s < \frac{N-1}{N} \) with Identical Status. This covers the entire range of values estimated for \( \beta_2 \beta_1 + \beta_2 \) from equation (5).\(^{11}\)

One of the problems in the literature is identifying the reference groups (e.g. the relative status weights \( \omega_{i,j} \)) and the correlates for the intensity of status comparisons by individual (e.g. the \( s_i \)). Only a few studies have direct evidence from individuals. In Knight et al. (2009), Chinese villagers report comparing to neighbors and not much beyond their village. The intensity of comparisons decreases with own income. Clark and Senik (2010) examine an European Social Survey of 18 countries which asks respondents to whom do you compare yourself as well as how intensely. Comparisons are mainly with people in proximity, family, friends and colleagues. They find strong evidence that individuals predominantly compare upwards towards those richer than themselves and that the intensity of comparisons decreases with own income.

In terms of our model, this evidence suggests that those with a higher income rank have smaller absolute weights \( s_i \). It also suggests that each agent \( i \) places a greater relative weight \( \omega_{i,j} \) on an agent \( j \) the greater is \( j \) in the income rank. We assume that these are the most relevant status specifications. They fit with anecdotal accounts going back to Veblen (1899) and Duesenberry (1949) as well as the success of using neighbors or similar socio-economics reference groups in empirical work. They are also consistent with the evidence from the positional concerns literature studied below.\(^{12}\)

### 3.2 Positional Concerns

The existence of positional concerns has been vividly revealed by experimental surveys. In their widely cited experiment, Solnick and Hemenway (1998) ask individuals to choose between two worlds, one where “your current yearly income is $50,000; others earn $25,000”, and another where “your current yearly income is $100,000; others earn $200,000.” An individual is said find evidence for relative income negatively influencing happiness, but only for a measure of hedonic well-being, and not for a measure of evaluative well-being.

\(^{11}\)See Table 5 in Appendix C for the solution to the utility equation (4) in the case of Identical Status. In equation (5) the reference group benchmark is the logarithm of average income, \( \ln y^* \); whereas, in our model with log income it is the average of log incomes, 
\[ U_i^* = \frac{1}{N-1} \sum_{j \neq i} \ln y_j. \]
Only if \( y_j = y^* \) for all \( j \) can we directly back out an estimate of \( s \). In this case, a ratio \( \frac{\beta_2}{\beta_1 + \beta_2} = \frac{s}{2} \) would imply \( s = \frac{1}{4} \) for \( N \) large.

\(^{12}\)Corazzini et al. (2012) find that comparisons in positional experiments are made not only upwards and but also downwards. Relative income concerns are far greater in high-income countries than low-income countries.
to have a positional preference for income if she prefers the former world, which is often the case. However, when the same questions are about leisure, offering the choice between 2 weeks of vacation when others have 1 week versus 4 weeks when others have 8 weeks, individuals typically prefer the latter non-positional world.

Johansson-Stenman et al. (2002) measure positional concerns using utility specification \[ \tilde{U}(x, \bar{x}) = x^{1-\gamma} \left( \frac{x}{\bar{x}} \right) ^\gamma, \]
where \( x \) is own consumption, \( \bar{x} \) is others’ per capita consumption, and the exponent \( \gamma \in [0, 1] \) is interpreted as how good \( x \)’s level of positionality is. Good \( x \) is non-positional when \( \gamma = 0 \) and positional with full offset when \( \gamma = 1 \). Denote the positional and absolute distributions used in surveys, \( ((x_p, \bar{x}_p), (x_a, \bar{x}_a)) \), with \( \bar{x}_a > x_a > x_p > \bar{x}_p \). A consumer is said to have “level of positionality” \( \gamma_x \) for good \( x \) if she is indifferent between the two distributions, \( \tilde{U}(x_a, \bar{x}_a) = \tilde{U}(x_p, \bar{x}_p) \). The median range for the level of positionality for income reported in Johansson-Stenman et al. (2002) is \( \gamma_x \in [0.2, 0.5] \), which has been confirmed in subsequent studies (Alpizar et al. 2005, Carlsson et al. 2007). These studies also estimate \( \gamma_x \) over a variety of goods and find significant differences in the levels of positionality across goods. Carlsson et al. (2007) cannot reject \( \gamma_x = 0 \) for leisure, and Solnick and Hemenway (2005) conclude that leisure is a much less positional good than income.

Our model can parsimoniously explain different positional concerns without requiring different positional coefficients (\( \gamma \)) for each good. Primary utilities depend only on the level of \( x \), and the positional and absolute distributions respectively give \( (U(x_p), U(\bar{x}_p)) \) and \( (U(x_a), U(\bar{x}_a)) \). Using these primary utilities in the utility system, \( u(x, \bar{x}) = b_{i,i}U(x) - (b_{i,i} - 1)U(\bar{x}) \), an agent prefers the positional over the absolute distribution if and only if \( u(x_p, \bar{x}_p) > u(x_a, \bar{x}_a) \) or

\[
\frac{b_{i,i} - 1}{b_{i,i}} > \frac{U(x_a) - U(x_p)}{U(\bar{x}_a) - U(\bar{x}_p)}
\]

Here \( \frac{b_{i,i} - 1}{b_{i,i}} \in (0, 1) \) as \( b_{i,i} > 1 \). Thus, the positional distribution is preferred when \( U(x_a) - U(x_p) \) is small relative to \( U(\bar{x}_a) - U(\bar{x}_p) \); that is, the own primary utility gain is small relative to others’ primary utility gain. The condition is satisfied as \( x_p \) approaches \( x_a \), and it fails as \( x_a - x_p \) approaches \( \bar{x}_a - \bar{x}_p \).

To explain the example from Solnick and Hemenway (1998), we show that the above condition can be satisfied by income while at the same time failing for vacations. An individual prefers the positional distribution for income (interpreted as consumption on the RHS) and the absolute distribution for vacations (interpreted as leisure on the LHS) if

\[
\frac{U(c, 4) - U(c, 2)}{U(c, 8) - U(c, 1)} > \frac{b_{i,i} - 1}{b_{i,i}} > \frac{U(100000, l) - U(50000, l)}{U(200000, l) - U(25000, l)}
\]
where the primary utility function is $U(c, l)$. It is convenient to assume that $U(c, l)$ is additively separable so that the endowment $(c, l)$ does not affect the choice. $^{13}$ Using the benchmark Identical Status case gives the specific threshold $\frac{b_{i,i}^{-1}}{b_{i,j}} = \frac{(N-1)s}{N-1-s}$. Now the inequalities hold for many values of $(s, N, U(., .))$ since the only requirement on the parameters with Identical Status is $s < \frac{N-1}{N}. {14}$ Here the survey evidence is consistent with invidious utility comparisons without resort to asymmetrical positional preferences over goods.

4 Equilibrium, Efficiency, Waste, and Voluntary Transfers

Consider an environment where each agent $i$ takes prices, wages and the allocations of others as given. In a standard economy, where status is not considered ($s_i = 0$ for all $i$), each agent $i$ maximizes primary utility $U(c_i^*, l_i^*)$, where $(c_i^*, l_i^*)$ is the primary utility maximizing allocation. Nonsatiation (which is implied by $U$ increasing in $c$ and $l$) ensures the budget constraint binds. Similarly, in our model, agent $i$ maximizes utility, equation (4)

$$u(c_i, l_i) = b_{i,i} U(c_i, l_i) + \sum_{j \neq 1} b_{ij} U(c_j, l_j)$$

subject to the budget constraint. As Assumption 1 ensures that $b_{ii} > 0$, agent $i$ maximizes utility, $u(c_i, l_i)$, by maximizing her primary utility, $U(c_i^*, l_i^*)$. The agent’s choice is unaffected by the parameters of the social utility function $(s_i, \omega_{i,j})$. This result seems counter-intuitive: agent $i$’s choices influence the utilities of other agents through the status equation (2), which in turn influences agent $i$’s utility. Thus, it seems that status concern should affect agent $i$’s decisions. And it does, but without generating distortions since other agents only care about agent $i$’s utility which is an increasing function of primary utility. Thus, the best agent $i$ can do is to maximize her primary utility. In order to formalize this equivalence result, assume a competitive equilibrium exists and the First and Second Fundamental Theorems of Welfare Economics are satisfied in the economy where agents do not consider status ($s_i = 0$ for all $i$).

**Proposition 2.** The set of equilibrium and efficient allocations are the same in an economy in which some agents consider status ($s_i > 0$ for at least one agent $i$) and in an otherwise identical economy where agents do not consider status ($s_i = 0$ for all $i$).

$^{13}$In the literature, respondents are asked to choose between worlds where only one good is varied without mentioning other goods. When goods are additively separable in utility function, the amounts of other goods is irrelevant.

$^{14}$For example, the inequalities hold when $N = 1000$ for $U(c, l) = c + \ln(l)$ with $s \in (0.286, 0.333)$, or for $U(c, l) = \frac{c^{\eta-\eta-1}}{\eta-\eta} + \max[4 - l, 0]$ for any $0 \leq \eta < 4$ with $s > \frac{1}{4}$. 

14
Competitive equilibrium economies with and without status utility are not only observationally equivalent but also have the same efficiency implications. As in standard policy analysis, lump sum taxation can be used to shift the competitive equilibrium efficient outcome to address equity concerns. Implications for the distribution of utility are discussed in Section 5.3. Efficiency equivalence fails when we relax the assumption of primary utility nonsatiation as discussed in Section 4.2.

Proposition 2 adds to existing results. In contrast to our model, with altruistic preferences the First Welfare Theorem may fail. With altruism, an allocation where one agent has everything is a competitive equilibrium, but is not optimal as all may agree that a small transfer to others is Pareto improving. Interestingly, with status utility the First Welfare Theorem holds despite the possibility of each agent’s utility being positively affected by a subset of others, i.e. $b_{i,j} > 0$ for a subset of $j$ for each $i$. Moreover, the neutrality result in Proposition 2 is related to papers finding efficiency with a status market or with envy.

### 4.1 The Invidious Comparison - Efficiency Paradox

The efficiency consequences of invidious comparisons are difficult to pin down on theoretical or empirical grounds. As discussed in the introduction, the status literature largely presumes that intense status comparison should inevitably lead to large allocative distortions. Yet, Arrow and Dasgupta (2009) do not find that the empirical literature directly supports this presumption. Their explanation is that allocative distortions cancel out when all goods are positional with symmetric offsetting externalities. In contrast, our model not rely on positional preferences over goods while allowing substantial asymmetries in positional concerns.

Of course, very strong positional concerns on some visible goods (Heffetz 2011, Charles et al. 2009) such as luxury goods (Frank 2010) and cars (Grinblatt et al. 2008, Kuhn et al. 2011, Winkelmann 2012) is hard to comprehend without appealing to positional preferences on select goods. We acknowledge that our model is not the right one for these goods, but the key issue is whether distortative externalities are limited to a small fraction of the economy. The major margins for potentially large distortions are around labor-leisure and inter-temporal savings choices. Clark et al. (2008) indicate that leisure or labor should be examined, as a

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15 See Archibald and Donaldson (1976) for a summary. When primary utilities are interdependent and possibly spiteful, Rader (1980) derives a general version of the Second Welfare Theorem assuming “no local Pareto satiation” and Dufwenberg et al. (2011) show that the set of efficient allocations is a subset of the efficient allocation without. Note that we obtain a strong equivalence result in our application, and this is because the transformation matrix $A$ implies a tight connection between primary utilities and utilities.

16 Becker et al. (2005) examine a world where status is a good that can be traded but not produced. When the status good is a complement with consumption, it will be efficiently traded. Kolm (1995) characterizes conditions on initial allocations that neutralize the impact of envy on the final allocation.
third argument in an extension of equation (5), as evidence for losses from excess labor supply. However, they do not cite, nor were we able to find, any estimates of welfare losses in that literature.\textsuperscript{17} It is difficult to estimate such losses given the complexity of labor supply and the many characteristics that workers value (Clark et al. 2012).

The case for saving distortions is stronger, although not definite. After taking into account endogenous reference group effects, Maurer and Meier (2008) find modest externalities in their lifecycle analysis of PSID data. Alvarez-Cuadrado et al. (2015) also estimate a lifecycle model. Using Spanish data, they find a substantial consumption externality when the reference group is the census tract but find no externality with a socio-economic reference group (as in Maurer and Meier (2008)). Bertrand and Morse (2013) and Frank et al. (2014) examine US data and find evidence for “expenditure cascades”, also called “trickle down effects”, whereby increases in expenditures (or earnings) of those at the top of the income distribution increases the expenditures of those below. Direct evidence for widespread allocation distortions along important dimensions is sensitive to the specification of reference groups and the difficulty of establishing causation and ruling out other hypotheses. Without more analysis, the puzzle of intense invidious comparisons without necessarily large allocation distortions remains. We leave the debate aside and focus in Section 5 on the issue of (non-distortative) welfare consequences of invidious comparisons which has been surprisingly untouched in the literature.

4.2 “Waste” from Insatiable Status

A major insight in economics is that selfish agents interacting in competitive markets may achieve an efficient outcome. Proposition 2 shows that this result generalizes to status conscious agents. Here we show that efficient outcomes may continue to obtain even if a subset of agents are sufficiently rich to be satiated in own primary utility, i.e. satiated in both consumption and leisure. Efficiency requires behavior that seems wasteful because it involves either consumption beyond satiation or the disposal of resources.\textsuperscript{18} This miserly behavior is the consequence of the maintained assumption that agents are never satiated in status utility. Now, the equivalence between models as described in Proposition 2 fails, because efficiency in the standard model without status involves distributing some goods to those agents who are not satiated.

Consider the Two Agents example, but where agent 2 is satiated in primary utility but not status utility. Then, the frontier of the primary utility possibility set, in the positive quadrant, is flat up to the point where agent 2 stops being satiated. With $s_2 > 0$ we have $b_{2,1} < 0$

\textsuperscript{17}Alvarez-Cuadrado (2007) offers calibration evidence of substantial inefficiencies assuming that one-half of consumption has no effect because it is status related.

\textsuperscript{18}Brennan (1973) finds that waste may be efficient with envy, although never unilaterally.
so that the frontier of the utilities possibilities set is downward sloping. Any allocation that achieves a point on the flat segment of the primary utility frontier corresponds to a point on the downward sloping utility frontier. Thus, consuming beyond the threshold for primary utility satiation, increases the utility of agent 2 as it deprives agent 1 of consumption. The example generalizes to costly disposal and many agents. Agent 2 has a strong incentive to deprive agent 1 from consuming. With diminishing marginal primary utility, the marginal negative effect of agent 1 on agent 2 is greatest when agent 1 is consuming least. Thus, agent 2 would be willing to incur at least a small cost to dispose of extra resources to deprive agent 1 of them. But, such willful waste by standard sensibilities can be thought of as “conspicuous consumption” and “conspicuous waste”. Willful waste could take the form of lavish gift-giving and partying as long as the activity were strictly in the company of the consumption satiated.

Gift giving becomes spiteful when a consumption satiated agent $i$ benefits from a subset of other agents $j$ doing well, i.e. $b_{i,j} > 0$. At the margin there is zero cost to giving to agents $j$. Thus, some resources are gifted to at least one non-satiated agent $j$. It is easy to imagine that gifts to the enemy-of-my-enemy might resemble Veblen’s “vicarious consumption” where, for example, paying associates and underlings extravagantly invokes the envy of rivals and their entourages. Whereas satiable consumption is sufficient for gift giving to indirectly spite others, such behavior can also occur even with nonsatiation as we analyze next.

4.3 Spiteful Transfers

In the standard model, without status utility, the core of the economy is equivalent to the set of competitive allocations. However, this core-equivalence result may fail in our model when a subset of agents can increase all members utilities through a redistribution among themselves. Since no Pareto improvement is possible, the utility of at least one agent in the rest of the economy has to decrease. We assume nonsatiation so that the marginal rate of transformation along the primary utility Pareto frontier is positive, $MRT_{i,j}^U = \frac{dU_i}{dU_j} \bigg|_{PFU} > 0$.

Consider a transfer from one agent, agent $i$, to another agent, agent $j$. Equation (4) identifies the cross primary utility benefit to agent $i$ of agent $j$ receiving a unit primary utility transfer, $dU_j = 1$, to be $b_{i,j}$. Agent $i$ gives only if $b_{i,j} > 0$; she is able to spite others by making agent $j$ better off (see Section 3 and footnote 6). The cost for agent $i$ to make such a transfer is $b_{i,i}MRT_{i,j}^U$. Thus, agent $i$ is better off giving when $b_{i,j} > b_{i,i}MRT_{j,i}^U$. This condition and the following definition motivate the proposition below. From equation (4), define agent’s $i$ marginal rate of substitution between own and agent $j$’s primary utility, $MRS_{j,i} = \frac{\frac{dU_j}{dU_i}}{\frac{dU_j}{dU_i}} = \frac{b_{i,j}}{b_{i,i}}$.

**Proposition 3.** A mutually agreeable transfer of resources from agent $i$ to agent $j$ exists if and only if $MRT_{j,i}^U < MRS_{j,i}$.
The inequality \( MRT_{j,i}^U < MRS_{j,i} \) says that agent \( i \) will give a gift when the feasible rate of transferring primary utility is less than the subjective threshold rate that makes agent \( i \) no worse off. The inequality is satisfied when \( MRT_{j,i}^U \) is sufficiently small and \( b_{i,j} > 0 \) so that agent \( i \) is able to spite others outside the group by agent \( j \) doing better. The inequality is only satisfied if \( MRT_{j,i}^U < 1 \), since \( MRS_{j,i} < 1 \) from \( b_{i,i} > b_{i,j} \) in Proposition 1(ii). Thus, if all agents have the same primary utility functions, agent \( i \) would have to be richer than agent \( j \). There would be no bilateral transfers in an egalitarian society.

We now show that multilateral redistributions can readily occur between agents within a group of any size. Consider the Two Mutually Envious Groups economy in Table 1, where the size of the first group is \( n < N \). Let \( n \) be an even number so that we can divide \( \frac{n}{2} \) with i’s in one subgroup and j’s in the other subgroup. Suppose that each pair \((i, j)\) has the same \( MRT_{j,i}^U < 1 \). Then an equal transfer \( dU_i = -MRT_{j,i}^U dU_j < 0 \) from \( i \) to \( j \) for all pairs \((i, j)\), increases the representative agent \( i \)’s utility if and only if the benefit exceeds the loss. The benefit is \( \frac{n}{2} MRS_{j,i} \), where \( MRS_{j,i} = \frac{s^2}{s^2 + n(1 - 2s)} \). There is a direct loss from \( i \) giving \( MRT_{j,i}^U \) and an indirect loss associated with the other \( \frac{n}{2} - 1 \) givers. Overall, the representative giver benefits if \( \frac{n}{2} MRS_{j,i} > MRT_{j,i}^U (1 + (\frac{n}{2} - 1)MRS_{j,i}) \) and after simplifications

\[
\frac{MRT_{j,i}^U}{1 - MRT_{j,i}^U} < \frac{s^2}{2(1 - 2s)}
\]

This condition obtains for \( MRT_{j,i}^U \) small enough, independent of group size \( n < N \). The redistribution reduces both primary utility and utility inequality within the group by spiting those outside the group who suffer a status loss of utility.

5 Welfare

This section explores welfare implications of status that are not due to allocative inefficiencies, an area that has been largely overlooked by the literature. By Proposition 2, allocations are unaffected by status utility. Thus, status parameters \((s, \omega)\) do not affect primary utility and can only affect utility through status utility, as \( u_i = U_i + S_i \). We focus on four key issues:

(1) When does status decrease average utility in society? (2) Can this offer an explanation for the Easterlin paradox? (3) When does status increase inequalities amongst individuals? (4) What redistributive policies increase/maximize average utility, and what are the implications for equity and status? Without loss of generality, we assume that an initial allocation is given and order the primary utility of agents \( U_1 \leq U_2 \leq ... \leq U_N \). We describe agents \( i = 1...N \) as ordered from the least to the most affluent, to loosely make the connection to the well-being
literature where the challenge is to explain happiness and income.

5.1 Average Utility and Negative Sum Status

We start by looking at average utility as there is a major policy interest in explaining average well-being in society. Denote average utility \( \bar{u} \equiv \frac{1}{N} \sum_{i} u_i \). Average utility can be written \( \bar{u} = \bar{U} + \bar{S} \), where \( \bar{U} \equiv \frac{1}{N} \sum_{i} U_i \) is average primary utility and \( \bar{S} \equiv \frac{1}{N} \sum_{i} S_i \) is average status utility. Status parameters \((s, \omega)\) only affect \( \bar{u} \) through \( \bar{S} \). For example, “negative-sum status” \( \bar{S} < 0 \), implies a loss of utility compared to primary utility \( \bar{u} < \bar{U} \). A useful benchmark is when zero-sum status \( \bar{S} = 0 \) obtains for any distribution of primary utility.

**Definition 1.** Utility conservation holds if \( \bar{u} = \bar{U} \), or equivalently \( \bar{S} = 0 \), for any vector \( U \).

To identify zero-sum status situations, denote by \( \lambda_i \equiv \sum_{k} b_{k,i} \) the column-sum of matrix \( B \). It is agent \( i \)’s “marginal contribution to total utility” since \( \lambda_i = \sum_{k} \frac{\partial u_k}{\partial U_i} = N \frac{\partial u_i}{\partial U_i} \). Similarly, denote \( \tilde{w}_i \equiv \sum_{k \neq i} \omega_{k,i} s_k - s_i \). It is related to the column-sum of matrix \( A \). When \( s_i \) is constant, \( \tilde{w}_i = s (w_i - 1) \) where \( w_i \equiv \sum_{k \neq i} \omega_{k,i} s_k \) is a measure of the “cumulative relative status envy” that society feels toward agent \( i \). Agents with high \( w_i \) attract more status envy from others ceteris paribus. Hence, \( \tilde{w}_i \) can be thought of as the “weighted cumulative status envy” that society feels towards agent \( i \). Though there is no uncertainty in the model, covariances are useful to characterize average status utility.

**Lemma 2.** \( \bar{S} = -Cov (\tilde{w}_i, u_i) = Cov (\lambda_i, U_i) \).

Zero-sum status, \( \bar{S} = 0 \), obtains if either \( \tilde{w}_i \) or \( \lambda_i \) are constant for all \( i \). The following proposition describes the specific relationship.

**Proposition 4.** The following three statements are equivalent: (i) Utility conservation, (ii) \( \tilde{w}_i = 0 \) for all \( i \), and (iii) \( \lambda_i = 1 \) for all \( i \).

Utility conservation follows for Identical Status societies as \( s_i = s \) and \( \omega_{i,j} = \frac{1}{N-1} \) give \( \tilde{w}_i = 0 \). More generally, when \( s_i = s \), utility conservation requires the same total envy be directed toward all agents, which is implied, for example, by \( \omega \) symmetric. When the absolute status intensities vary, utility conservation requires \( s_i = \sum_{k \neq i} \omega_{k,i} s_k \), so that agents with high \( s_i \) put a greater relative status weight on other agents who also have high absolute status intensities.

Lemma 2 also allows us to identify some general patterns for \( \tilde{w}_i \) and \( \lambda_i \) that determine the sign of \( \bar{S} \). Negative-sum status obtains if agents’ marginal contributions to total utility \( (\lambda_i) \) decrease as their primary utilities \( (U_i) \) increase. Negative-sum status \( (\bar{S} < 0) \) also obtains if the
weighted cumulative status envy that society feels towards agents \((\tilde{w}_i)\) increases with the utilities of agents \((u_i)\). Whereas primary utilities are assumed to be weakly increasing in \(i\), utilities are endogenous and \(u_{i+1} \geq u_i\) does not necessarily obtain. Section 5.3 derives a sufficient condition for the utility ranking to be preserved which is slightly stronger than \(\tilde{w}_i\) increasing. The following proposition also shows that, when agents have identical relative status intensities \((\omega_{i\neq j} = \frac{1}{N-1})\), the sign of \(\overline{S}\) can still be determined even if the ranking of \(u_i\) is not preserved.\(^{19}\)

Proposition 5. \(\overline{S} \leq 0\) if any of the following three statement holds for all \(i < N\): (i) \(\tilde{w}_{i+1} \geq \tilde{w}_i\) and \(u_{i+1} \geq u_i\), (ii) \(s_{i+1} \leq s_i\) and \(\omega_{i\neq j} = \frac{1}{N-1}\), or (iii) \(\lambda_{i+1} \leq \lambda_i\) and \(U_{i+1} \geq U_i\).

The literature reviewed in Section 3.1 suggests that evidence about absolute and relative status intensity is at odds with utility conservation. First, the less affluent tend to compare more intensely which suggests that absolute status intensities \(s_i\) are decreasing with \(i\). Then \(\tilde{w}_i\) are increasing in \(i\) ceteris paribus (i.e. for identical relative status intensities, \(\omega_{i,j} = \frac{1}{N-1}\)). Second, individuals tend to compare themselves more intensely to those who are more affluent (\(w_{i,j}\) increases with \(j\)). This pattern, which suggests that richer agents attract greater cumulative relative status envy, implies that \(\tilde{w}_i\) are increasing in \(i\) ceteris paribus (i.e. holding absolute status weights constant across agents, \(s_i = s\)). Since \(\tilde{w}_i\) are increasing when \(s_i\) are decreasing or \(w_i\) are increasing ceteris paribus, status is negative sum in societies where poorer individuals envy more or where richer individuals tend on average to be envied more.

5.2 The Easterlin Paradox Revisited

A novel result from our model is that an increase in everyone’s primary utility in the sense of stochastic dominance may have a negligible or even negative impact on average utility. This is illustrated in Figure 1a, which plots the values of \(\lambda_i\) for four parametrized economies detailed in Table 2. In the case of Two Mutually Envious Groups, increasing the utility of any of the top three agents reduces average utility \((\lambda_i < 0\) for \(i \geq 8\)! With All-up Status, increasing the primary utility of the most affluent agent has a negligible impact on average utility \((\lambda_{10} = .08 \ll 1\)."

To illustrate the cross-sectional impact of increasing the affluence of the top cohort on the happiness of each cohort, Figure 1b plots \(b_{i,N} \equiv \frac{du_i}{dU_N}\) for all \(i\), where the top agent is \(N = 10\). Consistent with Proposition 1, the impact on the top agent is positive (own effect \(b_{N,N} \geq 1\)) and strongest in magnitude \((b_{N,N} > |b_{i,N}|\) for \(i < N\)). Also, the utility inequality between the rich and the rest increases \((b_{N,N} - b_{i,N} \geq 1\) typically). Beyond this, the figure reveals complex patterns on the happiness in the rest of society. With Identical Status all other agents are hurt

\(^{19}\)See Appendix C for an example with identical relative status intensities where the ranking is not preserved.
Figure 1: $\lambda_i$ and $b_{i,N}$ for Four Benchmark Status Functions

(a) $\lambda_i$ for $i = 1..10$

(b) $b_{i,N}$ for $i = 1..10$

Note: The figures assume $N = 10$, $s = .4$.

Table 2: Marginal Contributions ($\lambda_i$) and Average Status ($\bar{S}$)

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_i$</th>
<th>$\bar{S} &lt; 0; \bar{S} = 0; \bar{S} &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Agents</td>
<td>$(1-s_1)(1-s_2)-s_1 s_2$</td>
<td>$s_1 &gt; s_2; s_1 = s_2; s_1 &lt; s_2$</td>
</tr>
<tr>
<td>Identical Status</td>
<td>1</td>
<td>Zero Sum</td>
</tr>
<tr>
<td>2-Mutually Envious Groups</td>
<td>$(1-2s)^{-1} \left(1 - \frac{s_1}{1-\alpha}\right)$ for $i &gt; n$</td>
<td>$n &gt; \frac{1}{2}; n = \frac{1}{2}; n &lt; \frac{1}{2}$</td>
</tr>
<tr>
<td>All-up Status</td>
<td>$\lambda_n = \lambda_{n-1} \left(1 - \frac{s}{N-n-1}\right)$</td>
<td>Negative Sum</td>
</tr>
</tbody>
</table>

a little. The One-up Status and All-up Status cases display ripple effects. With Two Mutually Envious Groups there is a trickle down pattern (Bertrand and Morse 2013).

These examples offer a new explanation for the Easterlin Paradox (Clark et al. 2008). The income growth in the past few decades has largely benefited the richest in society and has given rise to widespread discontent among the non-rich suggestive of happiness losses and magnified happiness inequality. In the examples above, income growth at the top can be associated with a negligible or negative impact on average happiness and with no reduction in the inequality of happiness. This negative status externality analysis does not rely on features like adoption, aspirations, or concavity of utilities, and most importantly, does not imply allocative inefficiencies. Unlike the relative income hypothesis, our analysis relies on status preference asymmetries and yields average happiness gains without them. It is also a caution against the common practice of averaging income and happiness within a group or country which ignores potentially important
5.3 Status-Induced Inequality

Figures 2 and 3 plot the distribution of utility for $s = 0.4$, for a given sets of weights $\omega$ and for given vectors of primary utilities. Table 3 summarizes some of the properties displayed in these figures that hold more generally. Taken together, the table and figures show that the distribution of primary utility can be transformed into a rich set of utility distributions.

**Figure 2: Primary Utility ($U$) and Utility ($u$)**

![Graph showing the distribution of utility for Identical and Heterogenous weights.](image)

Note: The blue line plots the values of primary utilities and the green line plots the values of utilities. These figures assume $N = 10$ and that primary utility takes values $U_i = i$ for $i = 1,..,10$. Figure (a) assumes $s = .4$ and figure (b) assumes $s_1 = .4, s_{10} = 0$ and status weights decrease by equal increments.

Figure 2(a) plots the distribution of utility for Identical Agents. Two interesting patterns emerge: (A) The ranking of utilities is preserved ($u_i$ is increasing in $i$); and (B) Status magnifies utility inequalities (the utility curve is steeper than the primary utility curve). Properties (A) and (B) holds more generally when the weights $(\omega_{i,j})_{i,j}$ satisfy a monotonicity property that is defined shortly. The remaining figures illustrate violations of properties (A) and/or (B). Figure 2(b) assumes that $s_i$ is decreasing with affluence. This attenuates the impact of status and the attenuation effect increases with affluence; property (B) is violated. This property is also violated for One-up Status and All-up Status in Figure 3. In Figure 3 status magnifies inequalities in the case of Two Mutually Envious Groups and All-up Status. Property (A) is violated for One-up Status (right panel). Interestingly, that figure displays a saw tooth pattern.
where some agents move up in ranking while their neighbors move down.

Figure 3: Primary Utility ($U$) and Utility ($u$)

![Graphs showing utility changes for different scenarios](image)

Note: The blue line plots the values of primary utilities and the green line plots the values of utilities. $s = .4$ in all figures. Figures (a) and (b) set $N = 10$ and primary utility takes values $U_i = i$ for $i = 1,..,10$. In figure (a), there are two groups of 3 and 7, respectively. Figure (c) sets $N = 5$ and primary utility takes values corresponding to the 5 quintiles of the U.S. distribution of money income (2012 Census Bureau).

Table 3: Properties of $(U, u)$

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Magnify Inequalities</th>
<th>Order Preserved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Agents</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Identical Status</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>2-Mutually Envious Groups</td>
<td>Yes within groups</td>
<td>Yes</td>
</tr>
<tr>
<td>All-up Status</td>
<td>Not Always</td>
<td>Not Always</td>
</tr>
</tbody>
</table>

In order to examine the role of relative status intensities, we hold the absolute intensity weights fixed $s_i = s > 0$ for all $i$. Let $W_{i,j} \equiv \sum_{k \geq j} \omega_{i,k}$ denote the cumulated envy that agent $i$ feels toward those who are at least as affluent as agent $j$. Note that $W_{i,1} = 1$ and that a higher value of $W_{i,j}$ means that agent $i$’s reference group puts more weight on agents who are at least as affluent as agent $j$. The following monotonicity assumption orders cumulative envy.

**Assumption 2.** $s_i = s$ and $W_{i-1,j} \geq W_{i,j}$ for all $(i, j) > 1$.

This assumption is satisfied by societies with Two Agents, Identical Status, and Two Mutually Envious Groups for $n > N/2$.

**Lemma 3.** Under Assumption 2 the ranking of utilities is the same as the ranking of primary utilities: $u_1 \leq u_2 \leq \ldots \leq u_N$.

The inequalities are strict when $U_{i-1} < U_i$. The preserved ranking generalizes beyond Assumption 2. Say we compare two agents $i > j$ with identical reference groups $\omega_{i,k} = \omega_{j,k}$ for
For these two agents, we have \( b_{i,k} = b_{j,k} \) and \( b_{i,j} = b_{j,i} \). By Proposition 1 (ii) we have \( b_{i,i} > b_{j,i} \), and the utility order is preserved for these two agents.

\[
u_i - u_j = (b_{i,i} - b_{j,i})(U_i - U_j) \geq 0 \tag{7}\]

The utility order can be reversed when Assumption 2 is violated such that some relatively poor agents envy poorer agents more. For example, in Figure 3(c) the utility order is reversed with One-up Status and when the top agent is significantly above the rest of society.

**Corollary 2.** Under Assumption 2, utility inequalities are magnified:

\[
u_h - u_i \geq \frac{1}{1 - s} (U_h - U_i) > (U_h - U_i) \text{ for } U_h > U_i
\]

Identical Status yields the difference: \( u_h - u_i = \frac{N-1}{(N-1)(1-s)} (U_h - U_i) \). For \( N = 2 \) the multiplication factor, \( \frac{1}{1-2s} \), can be arbitrarily large. Interestingly, we get the same large multiplication factor \( \frac{1}{1-2s} \) with Two Mutually Envious Groups when all agents within each group have the same level of primary utility. The proof of Corollary 2, shows that \( u_h - u_i \) is composed of a “direct effect”, \( \frac{1}{1-2s} (U_h - U_i) \), which always magnifies utility differences and a “reference group effect”, \( -\sum_{j=2}^{N} (W_{h,j} - W_{i,j})v_j \), which is non-negative under Assumption 2. However, the reference group effect could be negative when Assumption 2 is violated as with One-up Status.

Finally, consider when an agent benefits from status comparisons. The utility definition \( u_i - U_i = s \left(u_i - \sum_{j \neq i} \omega_{i,j} u_j\right) \) implies that an agent \( i \) benefits from status utility if and only if \( u_i > \sum_{j \neq i} \omega_{i,j} u_j \). This occurs when agent \( i \) compares predominantly downward so that the relative status weights for agents with higher utility that herself are sufficiently small. The following statement directly follows from Lemma 3.

**Corollary 3.** Under Assumption 2, individual \( i \) is better off, \( u_i > U_i \), if her reference group is less affluent than herself, \( W_{i,i} = 0 \). Conversely, individual \( i \) is worse off \( u_i < U_i \), if her reference group is more affluent than herself, \( W_{i,i} = 1 \).

The Two Agents economy is a special case where agent 1 loses and agent 2 gains. In the case of Two Mutually Envious Groups, the less affluent group loses and the affluent group gains. The corollary holds even when Assumption 2 fails as long as the utility order is preserved. In the All-up Status example, all but the top agent are absolute losers.

---

20The results in the above corollary generalize beyond Assumption 2. Return to equation (7) which compares two individuals with identical reference groups. As long as \( b_{j,i} \) is not too positive, we have \( b_{i,i} - b_{j,i} > 1 \) and the difference in utility is magnified.
5.4 Maximizing Welfare

Status utility in general affects social optimal allocations. First consider the Utilitarian objective of maximizing the sum of utilities. Without status utility, the Utilitarian planner maximizes $\sum_{i=1}^{N} u_i = \sum_{i=1}^{N} U_i$ subject to being on the Pareto frontier in primary utility $PF^U$. The planner chooses a point on $PF^U$ which trades off primary utility at rate 1 between agents such that $MRT_{U,j,i} = \frac{dU_i}{dU_j} |_{PF^U} = 1$ for all pairs of agents $i$ and $j$. In contrast, in an economy with status utility, the planner maximizes $\sum_{i=1}^{N} u_i = \sum_{i=1}^{N} \lambda_i U_i$ subject to $PF^U$. Recall, $\lambda_i \equiv \sum_k \frac{\partial u_k}{\partial U_i}$ is individual $i$'s marginal contribution to total utility. Thus, the planner chooses a point on $PF^U$ that considers the external effects of each agent’s utility on other agents:

$$MRT_{U,j,i} = \frac{\lambda_j}{\lambda_i}$$

for all pairs of agents $i$ and $j$. Unless the marginal contributions of all agents to total utility are equal, $\frac{\lambda_j}{\lambda_i} = 1$ for all $i$ and $j$, status utility will affect the optimal allocation. This requirement of equal marginal contributions holds under utility conservation in Proposition 4. The following proposition shows that without utility conservation status utility matters.

**Proposition 6.** The Utilitarian optimal allocation is unaffected by status utility if and only if utility conservation holds.

When utility conservation does not hold, a Utilitarian planner allocates more to agent $j$ the greater is her marginal contribution $\lambda_j$. Consider the Two Agents economy with identical primary utility functions. The Pareto frontier $PF^U$ is symmetric around the 45 degree line. Without status utility, the Utilitarian planner picks the egalitarian outcome $U_j = U_i = U^*$ on $PF^U$. Now suppose agent $j$ compares to $i$ ($s_j > 0$) but agent $i$ does not compare to $j$ ($s_i = 0$). Then $\frac{\lambda_j}{\lambda_i} = \frac{1}{1 - 2s_j} > 1$, and the planner allocates $u_j > U_j > U^* > U_i = u_i > 0$. Ironically, it is the status conscious agent $j$ that is allocated more and achieves higher utility. Agent $j$ has a larger marginal contribution because she imposes less of a negative externality. The planner puts her on top so she can look down on the poorer agent $i$ and “count her blessings”. This generates positive-sum status utility, and average utility exceeds the egalitarian utility, $\bar{u} = \frac{u_i + u_j}{2} > U^*$. Note that implementing the Utilitarian allocation would involve reversing fortunes were we to start from an All-up Status economy where the less status conscious agent $i$ were initially richer.

Extreme inequality can be the consequence of incorporating status preferences into the Utilitarian calculus. Suppose all agents have identical primary utility functions, and consider

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21This allocation of primary utility can be implemented by an allocation of goods that is not explicitly modeled here. Recall that $PF^U$ is assumed to be downward sloping.
Two Mutually Envious Groups where agent 1 faces a group of all other agents $k \geq 2$. Then $\lambda_1 = \frac{1-N_s}{(1-s)}$. Let $s \geq \frac{1}{N}$, so that and $\lambda_1 \leq 0$, to focus on the corner solution where the planner allocates agent 1 zero consumption and hence zero primary utility $U_1 = U(0,0) = 0$. The impoverished agent 1 suffers negative utility as she compares herself with the majority; $u_1 = S_1 = -\frac{s}{(1-s)} U_k < 0$. Ironically, maximizing intergroup inequality gives positive-sum status utility, $\bar{S} > 0$, and average utility exceeds the egalitarian utility, $\bar{u} > U^*$. The affluent majority is better described as not counting their blessings so much as spitefully enjoying the poverty and enmity of the minority. This draconian prescription for inequality shows vividly that status preferences can lead to the classic equity critique of Utilitarianism (Sen 1973), where the Utilitarian criterion is criticized for unfairly allocating less resources to those whom are poorer (primary) utility generators. However, in our model, the inequality arises from the asymmetry in other regarding behavior.

We now consider a welfare criterion which places extra weight on the utilities of the less well off. In particular, consider the extreme case of a maximin planner who maximizes the minimum utility amongst all agents. The following proposition holds even when individuals do not have the same utility functions. As before, $U^*$ is the egalitarian primary utility on $PF^U$.

**Proposition 7.** Maximizing the minimum utility amongst all agents implies an egalitarian utility allocation $u_i = U_i = U^*$ for all $i$.

Unlike the Utilitarian allocation, the maximin allocation precludes status differences. This is optimal because the worst off agent does not benefit from status comparisons. Compared with allocations in economies which generate negative-sum status utility, the egalitarian utility allocation boosts average utility in the status component. It also weakly maintains the rank ordering in primary utility between any pair of agents. Moreover, it preclude conspicuous consumption and spiteful transfer behavior. These arguments for progressive taxation are different from, although complementary to, the case based on decreasing marginal utility of wealth.

6 Conclusion

This paper develops a pure hedonic theory that combines consumer utility, representing economic payoffs, with what we call status utility, capturing the social status payoff. An individual derives status utility from comparing her cardinal utility to others. Utility itself is taken as the logical metric for social status comparisons, because the hedonic approach takes maximizing utility as the ultimate end. We have argued that this is a logical starting point for economic theory and that this perspective is useful in understanding the empirical work on relative income and status comparisons.
The model is consonant with standard economic theory in ways that help explain puzzles associated with status, interpersonal comparisons, conspicuous and positional consumption, and relative income. Comparisons of utility do not change competitive equilibrium allocations or generate distortions. As we have reviewed, the empirical evidence is weak on status comparisons inducing large allocation inefficiencies. In contrast, there is very strong evidence that status comparisons have a negative impact on subjective wellbeing that largely offset the positive effects of more consumption. Our theory helps explain this invidious comparison - efficiency paradox without requiring offsetting symmetric externalities. It also reconciles survey evidence, which finds that positional concerns for consumption are more pronounced than for leisure, as plausibly generated by standard primary utility functions.

We attribute the discontent with status comparisons to the magnification of utility inequality and to the reduction in average utility. The increase in utility inequality is a general feature of our model, whereas average utility declines only when upward comparisons prevail as suggested by surveys we reviewed. This demonstrates that the standard assumption of identical status made in the literature on interpersonal comparisons is not innocuous. For example, we show that an increase in average income that accrues to the most affluent, does not have to be associated with an increase in average happiness as is commonly argued in the controversial debate on the Easterlin paradox. More generally, we show that status comparisons can have complex dispersion effects with trickle down and ripples across society. When comparisons are mostly upwards, policies that reduce inequality are welfare increasing. Our analysis is a caution and counterpoint to the focus in the status literature on the distortionary consequences of consumption externalities, and on the assumption that status comparisons are identical across agents.

In this paper, utility is upheld as the ultimate hedonic comparison. There is some direct evidence for hedonic comparisons but more investigation is needed. If nature or culture instrumentally pick status tournaments to minimize the allocative cost of status comparisons, then relative utility is a leading contender. The signaling of status through relative or positional consumption remains to be explored within asymmetric information extensions of our model. As culture and information change, so do individual responses and social norms. Facebook “friends” and Twitter “hearts” are examples of new media technologies that enable self-promotion and social comparison. Such social media appear to be leading rapidly changing social norms. Modeling cultures of conspicuous sociability as signaling “conspicuous happiness”, if you will, remains for future research.
References


van de Ven, N., and M. Zeelenberg. 2014. On the counterfactual nature of envy: It could have been me. *Cognition and Emotion* 1–18.


Appendices

A  Notation

Table 4: Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i = 1..N$</td>
<td>agents in the economy</td>
</tr>
<tr>
<td>$u_i$</td>
<td>agent $i$’s utility</td>
</tr>
<tr>
<td>$U_i$</td>
<td>agent $i$’s primary utility</td>
</tr>
<tr>
<td>$S_i$</td>
<td>agent $i$’s status utility</td>
</tr>
<tr>
<td>$s_i$</td>
<td>agent $i$’s absolute status intensity</td>
</tr>
<tr>
<td>$\omega_{i,j}$</td>
<td>agent $i$’s relative status intensity for agent $j$</td>
</tr>
<tr>
<td>$A$</td>
<td>matrix that transforms $U = Au$</td>
</tr>
<tr>
<td>$B$</td>
<td>matrix that transforms $u = BU$</td>
</tr>
<tr>
<td>$UPS^U$</td>
<td>utility possibility set in primary utility (same for $UPS^u$)</td>
</tr>
<tr>
<td>$PF^U$</td>
<td>Pareto frontier in primary utility (same for $PF^u$)</td>
</tr>
<tr>
<td>$MRT_{j,i}^U$</td>
<td>marginal rate of transformation $\frac{dU_i}{dU_j}$ along $PF^U$ (same for $MRT_{j,i}^u$)</td>
</tr>
<tr>
<td>$MRS_{j,i}$</td>
<td>$\frac{\partial U_i}{\partial U_j}</td>
</tr>
<tr>
<td>$\bar{S}$</td>
<td>$\bar{u} - \bar{U} = \frac{1}{N} \sum_i S_i$</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>$\sum_{j \neq i} \omega_{j,i}$</td>
</tr>
<tr>
<td>$W_{i,j}$</td>
<td>$\sum_{k \geq j} \omega_{i,k}$</td>
</tr>
</tbody>
</table>

B  Proofs

**Proof of Lemma 1:** (a) Recall $A \equiv [I - s + s\omega]$. Under Assumption 1, $a_{i,i} = 1 - s_i > \frac{1}{2} > 0$. This diagonal element is greater than the row sum of the off diagonal elements, $\sum_{j \neq i} a_{i,j} = s_i \sum_{j \neq i} \omega_{i,j} = s_i$. Thus, matrix $A$ is a non-negative matrix that is strictly row diagonally dominant with positive diagonal terms. Such matrices are part of the class of $P$-matrices (see Generating Method 4.1 in Tsatsomeros (2002)). $P$-matrices have positive principal minors. Thus, $\det A > 0$ and matrix $B$ is well defined. Let $A_{-i,-i}$ denote the matrix obtained by removing the $i^{th}$ row and column from $A$. We have $b_{i,i} = \frac{\det A_{-i,-i}}{\det A} > 0$ since $A$ has positive principal minors.

(b) Suppose Assumption 1 is violated by agents $(k,l)$ such that $1 - s_k - s_l \leq 0$. It is sufficient to show that for some matrix $\omega$: $b_{k,k} < 0$ or that it is not defined. Consider the weights $\omega_{k,l} = \omega_{l,k} = 1$. For these weights, $b_{k,k}$ is not defined for $1 - s_l - s_k = 0$ and for $1 - s_k - s_l < 0$, we have $b_{k,k} = \frac{1-s_l}{1-s_l-s_k} < 0$ since $s_l < 1$ by assumption. $\square$
Proof of Proposition 1: (i) Equation (10) in Ostrowski (1952) gives bounds

\[ b_{i,i} \in \left[ \frac{1}{|a_{i,i}| + t_i \sum_{j \neq i} |a_{i,j}|}, \frac{1}{|a_{i,i}| - t_i \sum_{j \neq i} |a_{i,j}|} \right] \]

where \( t_i = \max_{k \neq i} \frac{\sum_{j \neq k} |a_{k,j}|}{|a_{k,k}|} \). Since \( a_{i,i} = 1 - s_i \) and \( \sum_{j \neq i} |a_{i,j}| = s_i \), we obtain \( t_i = \max_{k \neq i} \frac{s_k}{1 - s_k} \) and

\[ b_{i,i} \in \left[ \frac{1}{1 - s_i + t_i s_i}, \frac{1}{1 - s_i - t_i s_i} \right], \; t_i \in (0,1) \]

under Assumption 1. If \( s_i > 0 \), \( b_{i,i} \geq \frac{1}{1 - s_i + t_i s_i} > 1 \). If \( s_i = 0 \), equation (2) gives \( u_i = U_i \).

(ii) Theorem 2.5.12, in Horn and Johnson (1991), states that if \( A \) is strictly row diagonal dominant then the inverse matrix \( B \) is strictly diagonally dominant of its column entries, so that \( b_{i,i} > |b_{j,i}| \) for all \( i \). (Note the different definitions for strictly row diagonally dominant and strictly diagonally dominant of its column entries).

(iii) We show that \( J = (1,...,1)^T \) is an eigenvector of \( A \) and \( B \) with eigenvalue one. For \( u = J \), we have \( A u = (I - s + s \omega)J = J - s J + s \omega J = J \). Thus, we obtain \( A J = J \). For \( U = J \), we have \( B U = B J = B A J = J \). □

Proof of Proposition 2: If \( b_{ii} > 0 \), then agent \( i \)'s optimization problem is an affine transformation of the primary utility problem, and the allocation is unaffected by status considerations.

We turn next to efficiency equivalence. Given our assumptions, the utility possibility set in primary utility, \( UPS^U \), is a convex set. Since the utility possibility set in utility, \( UPS^u \), is obtained from \( UPS^U \) by the linear transformation \( u = BU \), \( UPS^u \) is also convex. Let \( x = (x_1,..,x_N) \) denote an allocation, that is, \( x_i = (c_i, l_i) \). We say that \( x \) is associated with primary utility allocation \( U \) to mean \( U = (U(x_1),..,U(x_N)) \). Similarly, \( x \) is associated to utility allocation \( u \) where \( u = BU \). The sets of consumption allocations associated with Pareto frontiers \( PF^U \) and \( PF^u \) are respectively denoted \( X^U \) and \( X^u \). Since \( U() \) is increasing in its arguments, nonsatiation obtains, and \( PF^U \) touches the axis: \( PF^U = Cl(\partial UPF^U \cap (\mathbb{R}_+^N)) \).

We will prove that \( X^U \subseteq X^u \).

First we show \( X^u \supseteq X^U \). This result is proved by contradiction. Suppose \( x_0 \) is associated with \( U_0 \) and \( u_0 \) are such that \( U_0 \in PF^U \) and \( u_0 \notin PF^u \). We have \( u_0 = BU_0 \in UPS^u \). Statements \( u_0 \in UPS^u \) and \( u_0 \notin PF^u \) imply that there must exist a feasible redistribution \( r \) away from \( U_0 \) such that (a) \( U_0 + r \in UPS^U \), and (b) \( B(U_0 + r) > BU_0 \). From statement (a) and \( U_0 \in PF^U \), we obtain that \( r \) must have at least one negative element. Statement (b) simplifies to \( Br > 0 \). Multiply both sides of the inequality by the matrix \( A \). Since matrix \( A \) has positive diagonal and non-negative elements, \( ABr > 0 \) or \( r > 0 \). A contradiction.

Now consider \( X^u \subseteq X^U \). This is also proved by contradiction. Suppose \( x_0 \) is associated with \( U_0 \) and \( u_0 \) are such that \( U_0 \notin PF^U \) and \( u_0 \in PF^U \). Consider the deviation \( U_1 = U_0 + \epsilon_U \), where \( \epsilon_U = A \epsilon_u \) with \( \epsilon^T_u = (\epsilon, ..., \epsilon) \) for \( \epsilon > 0 \) small. Since \( A \) has non-negative elements and positive diagonal, we have \( \epsilon_U > 0 \). We argue next that \( U_0 \notin \partial UPF^U \). This is because (a) \( U_0 \notin PF^U \) and (b) \( u_0 \in PF^u \) implies that \( U_0 \notin \partial UPF^U \setminus PF^U \). Now \( U_0 \notin \partial UPF^U \) and \( UPF^u \) compact, imply that for \( \epsilon \) small enough \( U_1 \in UPF^U \). Corresponding to \( U_1 \) is \( u_1 \in UPF^u \) such that

\[ u_1 = BU_1 = B(U_0 + \epsilon_U) = BU_0 + BA \epsilon_u = BU_0 + \epsilon_u = u_0 + \epsilon_u > u_0 \]

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The results that \( u_1 \in UPF^u \) and \( u_1 > u_0 \) contradict the assumption \( u_0 \in PF^u \). \( \square \)

**Proof of Proposition 3:** For simplicity we do not model the change in good allocation but work directly with changes in primary utility. As shown in the text, agent \( i \) benefits from making a transfer if and only if \( MRT_{j,i}^U < MRS_{j,i} < 1 \). The recipient, agent \( j \), benefits if \( du_j = b_{j,j}dU_j + b_{j,i}dU_i > 0 \) or, equivalently, \( 1 > \frac{b_{j,i}}{b_{j,j}}(-\frac{dt_{j,i}}{dt_{j,j}}) \). This inequality holds as \( \frac{b_{j,i}}{b_{j,j}} < 1 \) by Proposition 1 (ii) and the transfer rate \( (-\frac{dt_{j,i}}{dt_{j,j}}) \leq MRS_{j,i} < 1 \). \( \square \)

**Proof of Lemma 2:** \( \bar{S} = \frac{1}{N}J'(u - U) = \frac{1}{N}J'(u - ABU) = \frac{1}{N}J'(I - A)u = -\frac{1}{N}J's(\omega - I)u \)
where \( J = (1, ..., 1)' \). Vector \( J's(\omega - I) \) has \( i^{th} \) element \( \bar{w}_i = \sum_k \omega_{k,i}s_k - s_i \) such that \( \sum_i \bar{w}_i = \sum_i \left(s_i - \sum_k \omega_{k,i}s_k\right) = 0 \). Thus, we obtain \( \frac{1}{N}J's(\omega - I)u = \text{Cov}(\bar{w}_i, u_i) \).

\( \bar{S} = \frac{1}{N}J'(u - U) = \frac{1}{N}J'(B - I)U = \frac{1}{N}(\lambda - J)'U = \text{Cov}(\lambda_i, U_i) \) where the last equality holds since \( \frac{1}{N}\sum_i \lambda_i = 1 \) (\( BJ = J \) gives \( J'BJ = J'J \) or \( \lambda'J = N \)). \( \square \)

**Proof of Proposition 5:** Statements (i) and (iii) follow directly from Lemma 2. We prove statement (ii). \( A \) is a matrix with elements \( a_{i,i} = 1 - s_i \) and \( a_{i,j} = \frac{b_{i,j}}{N - 1} \). We have \( \lambda_i = \sum_j b_{j,i} \) with

\[ b_{j,i} = (-1)^{i+j}\frac{|A_{i,j} - j|}{|A|} \]

where notation \( A_{i,j} \) stands the matrix obtained after removing line \( i \) and column \( j \) from \( A \) and to simplify the exposition we use notation \( \text{det}(M) = |M| \). After replacing the formula for \( b_{j,i} \), we obtain

\[ \lambda_i |A| = |A_{i,i} - i| + \sum_{j \neq i} (-1)^{i+j}|A_{i,j} - j|. \]

For \( i \neq j \), denote by \( \bar{A}_{i,j} \) the matrix obtained from matrix \( A_{i,j} \) as follows: (a) If \( j < i \), column \( i - 1 \) is moved after column \( j - 1 \). (b) If \( j > i \), column \( i \) is moved after column \( j - 1 \). Matrices \( \bar{A}_{i,j} \) have the same columns as matrices \( A_{i,j} \) and each line has identical non-diagonal entries. We have \( \bar{A}_{i,i+1} = A_{i,i+1} \) and \( \bar{A}_{i,i-1} = A_{i,i-1} \). Moreover, matrix \( \bar{A}_{i,j} \) satisfies the following property:

\[ |\bar{A}_{i,j}| = (-1)^{|i-j|-1}|A_{i,j}|. \]  

This property is obtained by applying the swap rule for determinants and noting that matrix \( A_{i,j} \) is obtained from matrix \( A_{i,j} \) after making \( |i - j| - 1 \) column swaps. Replace the
expressions $|A_{-i,-j}| = (-1)^{|i-j|-1} |\tilde{A}_{-i,-j}|$ in the equation for $\lambda_i |A|$: 

$$\lambda_i |A| = |A_{-i,-i}| - \sum_{j \neq i} |\tilde{A}_{-i,-j}|.$$ 

We evaluate the determinants in the above expression. In order to do so, we use the following result:

**Lemma 4.** (a) Let $M_k^N$ be a matrix such that $M_{i,i}^N = m_i$ for $i \neq k$ and $M_{i,j}^N = n_i$ for $i \neq j$ and for $i = j = k$. We have $|M_k^N| = n_k \Pi_{i \neq k} (m_i - n_i)$. (b) Let $M^N$ be a matrix such that $M_{i,i}^N = m_i$ and $M_{i,j}^N = n_i$ for $i \neq j$. We have $|M^N| = \Pi_i (m_i - n_i) + \sum_i n_i \Pi_{j \neq i} (m_j - n_j)$.

**Proof:** (a) Any line $l \neq k$ of $M_k^N$ can be written as

$$(n_i, ..., n_i, m_i, n_i, ..., n_i) = (n_i, ..., n_i) + (0, ..., 0, m_i, n_i, 0, ..., 0)$$

where $m_i$ and $m_i - n_i$ are on the $l^{th}$ column. Applying the line linearity rule of determinants, we obtain that the determinant of $M_k^N$ is equal to the sum of the determinants obtained by replacing line $l$ in $M_k$ by each of the two lines on the right hand side in the above expression. The first determinant is equal to zero because line $l$ is equal to $m_i(1, ..., 1)$ and line $k$ is equal to $m_i(n_i, ..., 1)$. Thus, $|M_k^N|$ is equal to the determinant of the matrix obtained by replacing line $l$ with $(0, ..., 0, m_i - n_i)$ in matrix $M_k^N$. We can repeat this operation for all lines but line $k$ and obtain that $|M_k^N|$ is equal to the determinant of $Q^N$ defined as: (a) $Q_{k,j} = n_k$ for $j = 1..N$, (b) for $i \neq k$, (b1) $Q_{i,k} = m_i - n_i$ and (b2) for $j \neq i$ $Q_{j,j} = 0$. Since $Q_{j,j} = 0$ for $i \neq j$ and $i \neq k$, $Q^N$'s determinant is equal to the product of its diagonal elements: $|Q^N| = n_i \Pi_{i \neq k} (m_i - n_i)$. (b) Line one of $M^N$ can be expressed as

$$(m_1, n_1, ..., n_1) = (m_1 - n_1, 0, ..., 0) + (n_1, ..., n_1)$$

Applying the line linearity rule of determinants to $M^N$, we obtain

$$|M^N| = (m_1 - n_1)|M^N-1| + |M_1^N|$$

where $M^N-1$ is obtained by deleting the first line and first column in matrix $M^N$. Applying result (a) we have $|M_1^N| = n_1 \Pi_{i \neq 1} (m_i - n_i)$. We can repeat the same decomposition to matrix $M^N-1$ to obtain

$$|M^N-1| = (m_2 - n_2)|M^N-2| + |M_2^N-1|$$

where $M^N-2$ is obtained by deleting the first two lines columns in matrix $M^N$. Applying result (a) we have $|M_2^N-1| = n_2 \Pi_{i \neq 1,2} (m_i - n_i)$. After replacement, we obtain the equation for $|M^N|$:

$$|M^N| = (m_1 - n_1)(m_2 - n_2)|M^N-2| + n_2 \Pi_{i \neq 2} (m_i - n_i) + n_1 \Pi_{i \neq 1} (m_i - n_i)$$

Repeat the same operation iteratively to obtain the expression $|M^N| = \Pi_i (m_i - n_i) + \sum_i n_i \Pi_{j \neq i} (m_j - n_j)$.

In order to simplify the expressions, we factor out $\frac{1}{N-1}$ in all matrices. For example, we write matrix $A$ as $\frac{1}{N-1}$ times a matrix with diagonal elements $(N - 1)(1 - s_i)$ and non-diagonal element $(i, j)$ equal to $s_i$. Applying Lemma 4 (a) to $|A_{-i,-i}|$, we obtain

$$|A_{-i,-i}| = (N - 1)^{-(N-1)} \left( \Pi_{j \neq i} (N - 1 + Ns_j) + \sum_{k \neq i} s_k \Pi_{l \neq k,i} (N - 1 - Ns_l) \right)$$

and applying Lemma 4 (b) to $|\tilde{A}_{-i,-j}|$, we obtain

$$|\tilde{A}_{-i,-j}| = s_j (N - 1)^{-(N-1)} \Pi_{l \neq j,i} (N - 1 - Ns_l).$$
These two expressions give the following identity

$$|A_{-i,-i}| = (N - 1)^{-(N-1)} \left( \prod_{j \neq i} (N - 1 - Ns_i) + \sum_{j \neq i} |\tilde{A}_{-i,-j}| \right)$$

and plugging this new expression for $|A_{-i,-i}|$ in equation (8) we obtain the new expression for $\lambda_i|A|

$$\lambda_i|A| = (N - 1)^{-(N-1)} \prod_{j \neq i} (N - 1 - Ns_j).$$

The first difference in $\lambda$ can thus be expressed as

$$\lambda_i - \lambda_{i+1} = \frac{N(N - 1)^{-(N-1)}}{|A|}(s_i - s_{i+1})\Pi_{j \neq i, i+1} (N - 1 - Ns_j).$$

Since $s_j < \frac{N-1}{N}$, when $s_i$ is decreasing (increasing) we obtain that $\lambda_i$ is decreasing (increasing) and deduct that $\bar{S} < 0$ ($> 0$) by applying Lemma 2. \[\square\]

**Proof of Lemma 3:** Define the difference in adjacent utilities $v_i \equiv u_i - u_{i-1}$ for $i > 1$ and let $v_1 \equiv u_1$. We first want to prove that $v_i \geq 0$ for $i > 1$. Note that $u_i = \sum_{k=1}^{j=N} v_k$. Subtracting $u_i = U_i + s(u_i - \sum_j \omega_{i,j}u_j)$ for agent $i$ and the same expression for agent $h > i$ gives

$$\sum_{j=1}^{j=N} (\omega_{h,j} - \omega_{i,j})u_j + \frac{1-s}{s}(u_h - u_i) = \frac{1}{s}(U_h - U_i)$$

(9)

where the first term on the LHS can be expressed

$$\sum_j (\omega_{h,j} - \omega_{i,j})u_j = \sum_j (\omega_{h,j} - \omega_{i,j})\sum_{k=1}^{k=j} v_k = \sum_{j=2}^{j=N} (W_{h,j} - W_{i,j})v_j$$

Note that the first term in the sum for $v_1$ drops because $W_{i,1} = 1$ for any $i$. For $h = i + 1$ equation (9) simplifies to

$$\sum_{j=2}^{j=N} \Delta W_{i+1,j}v_j + \frac{1-s}{s}v_{i+1} = \frac{1}{s}\Delta U_{i+1}; \text{ for } i = 1...N - 1.$$  (10)

where $\Delta U_{i+1} \equiv U_{i+1} - U_i$, $\Delta W_{i+1,j} \equiv W_{i+1,j} - W_{i,j} \in [-1,0]$ by Assumption 2. In matrix

$$b_{i,i} = (N - 1)^{-(N-1)} \prod_{j \neq i} (N - 1 - Ns_i) + \sum_{k \neq i} s_k\Pi_{j \neq k,i} (N - 1 - Ns_i)$$

$$b_{i,j} = -(N - 1)^{-(N-1)} \prod_{j \neq i} (N - 1 - Ns_j) + \sum_k s_k\Pi_{j \neq k,i} (N - 1 - Ns_i)$$

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notation we have
\[ (\Delta W + \frac{1-s}{s} I) v = \frac{1}{s} \Delta U \]

Here \( \Delta W \) is a \( N-1 \) square matrix with generic element \( \Delta W_{i+1,j} \in [-1,0] \). The matrix \( \Delta W + \frac{1-s}{s} I \) has the following properties: (a) non-positive off-diagonal elements \( \Delta W_{i+1,j} \) for \( j \neq i+1 \), (b) positive diagonal elements \( \Delta W_{i+1,i+1} + \frac{1-s}{s} > 0 \) since \( \frac{1-s}{s} > 1 \), and (c) positive column sums \( \sum_{i=1}^{N-1} \Delta W_{i+1,j} = W_{N,j} - W_{1,j} \in [-1,0] \).

Matrix \( \Delta W + \frac{1-s}{s} I \) is column diagonally dominant with positive diagonal entries and negative off-diagonal entries.

**Lemma 5.** Let \( z \) be a non-negative vector and \( C \) a square matrix with (a) non-positive off-diagonal elements, (b) positive diagonal elements, and (c) columns that sum to positive numbers. The solution \( v \) to system \( Cv = z \) is a non-negative vector.

Proof: Assume not; i.e. some elements \( v_i < 0 \) for \( i \in \Omega \subseteq \{1,2,...,N-1\} \). Sum all the lines \( i \in \Omega \):
\[
\sum_{i \in \Omega} v_j \sum_{j \in \Omega} c_{i,j} + \sum_{i \in \Omega} v_i \sum_{j \in \Omega} c_{i,j} = \sum_{i \in \Omega} z_i
\]
The properties of \( C \) imply that \( \sum_{j \in \Omega} c_{i,j} > 0 \) for \( i \in S \), as it includes diagonal elements. Since \( v_i < 0 \) for \( i \in S \), the first sum on the LHS is negative. Regarding the second sum, \( \sum_{j \in \Omega} c_{i,j} \leq 0 \) for \( i \notin S \), as it does not include diagonal elements. Since \( v_i \geq 0 \) for \( i \notin \Omega \), the second sum is non-positive, making the LHS < 0. The RHS > 0, a contradiction. \( \square \)

Applying Lemma 5 to equation (11) gives \( v \geq 0 \) or \( u_i \leq u_{i+1} \) for \( i < N \). \( \square \)

**Proof of Corollary 2:** Rewriting equation (9):
\[
u_h - u_i = \frac{1}{1-s}(U_h - U_i) - \frac{s}{1-s} \sum_{j=2}^{N} (W_{h,j} - W_{i,j})v_j.
\]
Assumption 2 requires \( W_{h,j} \leq W_{i,j} \) for \( h > i \). Thus, the second term in the above equation is non-positive and \( u_h - u_i \geq \frac{1}{1-s}(U_h - U_i) > U_h - U_i \) as \( \frac{1}{1-s} > 1 \) by Assumption 1. \( \square \)

**Proof of Proposition 6.** The optimality conditions from both planners’ problems are the same when \( MRT_{j,i}^U = \frac{\lambda_j}{\lambda_i} = 1 \) for all pairs \( i \) and \( j \). Together \( \lambda_i = \lambda_j \) for all \( i \) and \( j \) and \( \sum_{i=1}^{N} \lambda_i = N \) implies \( \lambda_i = 1 \) for all \( i \). By Proposition 4, \( \lambda_i = 1 \) for all \( i \) if and only if \( \tilde{S} = 0 \). \( \square \)

**Proof of Proposition 7.** The optimal allocation of a maximin planner is the point on \( PF^u \) that intercepts the 45 degree line. The proof of Proposition 1 Point (iii) shows that \( J \) is an eigenvector of \( A \) and \( B \). Thus, the allocation on \( PF^u \) that crosses the 45 degree line is also the allocation on \( PF^U \) that crosses the 45 degree line. Proposition 4 implies \( u_i = U_i \). \( \square \)

**C Examples**

Two Agents \( (N = 2) \): Equation (3) implies \( u_i = U_i + s_i(u_i - u_j) \) for \( i \neq j \in \{1,2\} \). We obtain \( b_{i,i} = \frac{1-s_i}{\det A}, b_{i,j} = -\frac{s_i}{\det A} \), and \( \det A = (1-s_i)(1-s_j) - s_is_j \). Under Assumption 1 \( (s_i \in [0,\frac{1}{2}) \)
for $i = 1, 2$), $\det A > 0$, $b_{i,j} > 0$ and $b_{i,j} < 0$.

**Identical Status** ($s_i = s$ and $\omega_{i,j} = \frac{1}{N-1}$): Matrix equation 3 simplifies to $u_i = U_i + s \left( u_i - \frac{1}{N-1} \sum_{j \neq i} u_j \right)$ for $i = 1, \ldots, N$. Solving for $u_i$ gives

$$u_i = \frac{N - 1 - s}{N - 1 - Ns} U_i - \frac{s}{N - 1 - Ns} \sum_{j \neq i} U_j$$

A necessary and sufficient condition for $b_{i,i} > 0$ is $s < \frac{N-1}{N}$. Then, all the cross effects are equal and negative: $b_{i,j} < 0$.

**Non-preservation of utility rankings under heterogeneous $s_i$:** Reconsider the 10-agent example in Figure 2(b) keeping $s_1 = .4$ and $s_{10} = 0$ but replacing equal declining increments with $s_i = .41 - .01i$ for $i \leq 9$. We obtain $u_9 = 11.01 > u_{10} = 10.00 > u_8 = 9.62 > \ldots > u_1 = -2.36$. Here agent 9 not only enjoys great benefits from counting her blessing over the less affluent but she also enjoys the utility comparison with the more affluent agent 10.

**Two Mutually Envious Groups:** $N \geq 3$ agents are divided into two groups. Assume $s_i = s$ for all $i$. The first group consists of agents $i \leq n$, where $2 < n < N$; members of this group are not envious of each other, $\omega_{i,j} = 0$ for $j \leq n$, but are equally envious of each member of the other group (their reference group), $\omega_{i,j} = \frac{1}{N-n}$ for $j > n$. Conversely, for the other group. For $i \leq n$, equation (3) simplifies to

$$u_i = U_i + s \left( u_i - \frac{1}{N-n} \sum_{j=n+1}^{N} u_j \right)$$

and we obtain $b_{i,i} = \frac{s^2 + n(1-2s)}{n(1-2s)(1-s)}$, $b_{i,j \leq n} = \frac{s^2}{n(1-2s)(1-s)}$, $b_{i,j > n} = -\frac{s}{(N-n)(1-2s)}$. (a) Consistent with Proposition 1, if $s < \frac{1}{2}$, then $b_{i,i} > 0$, $b_{i,j \leq n} > 0$, and $b_{i,j > n} < 0$. That is, agent $i$ benefits by other agents in their group ($j \leq n$) doing well. When others in their group are doing well, this hurts agents in the other group ($j > n$), which indirectly benefit's agent $i$. This result obtains even if $n = N - 1$, so that there is only one agent outside the group, agent $N$. In this case, only one of the cross effects is negative. (b) In general, for any pair of agents who are part of the first group, $(i,j) \leq n$ and $i \neq j$, we have $MRS^i_j = b_{i,j} b_{i,i} = \frac{s^2}{s^2 + n(1-2s)} > 0$. When $n = 2$, the condition in Proposition 3 simplifies to equation (6): \( \frac{MRT^U_j}{1-MRT^U_{j,i}} < \frac{s^2}{2(1-2s)} \).

**One-up Status:** Primary utilities are ordered such that $U_1 < U_2 < \ldots U_{N-1} < U_N$. Each agent $i < N$ only compares herself to the next richest agent, $\omega_{i,i+1} = 1$ and $s_i = s$. Equation (3) simplifies to $u_i = U_i + s (u_i - u_{i+1})$ for $i < N$. Agent $N$ is the richest agent and does not compare herself to others, $u_N = U_N$. Solving recursively, one obtains that the utility of agent $i < N$ depends only on the primary utilities of more affluent agents $k \geq i$:

$$u_i = \frac{U_i}{1-s} + \frac{1}{1-s} \sum_{k=i+1}^{N-1} \left( \frac{-s}{1-s} \right)^{k-i} U_k + \left( \frac{-s}{1-s} \right)^{N-i} U_N$$

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Table 5: Characterization of Utility Functions

<table>
<thead>
<tr>
<th>Condition</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Agents</td>
<td>( b_{i,i} = \frac{(1 - s_1)(1 - s_2) - s_1 s_2}{(N - 1)(1 - s) + (N - 2)s} )</td>
</tr>
<tr>
<td>Identical Status</td>
<td>( b_{i,j} = \frac{(1 - s_1)(1 - s_2) - s_1 s_2}{(N - 1)(1 - s) - s} )</td>
</tr>
<tr>
<td>Two Mutually Envious Groups</td>
<td>( b_{i,j} \leq n = \frac{s^2 + n(1 - 2s)}{n(1 - 2s) + (1 - s)} )</td>
</tr>
<tr>
<td>One-up Status</td>
<td>( b_{i,j} &gt; n = \frac{(1 - s_1)(1 - s_2) - s_1 s_2}{(N - 1)(1 - s) - s} )</td>
</tr>
<tr>
<td></td>
<td>( b_{i,j} = \frac{1}{1 - s}(-s)^{k-i}, b_{i,N} = \frac{(-s)^{N-i}}{1 - s} ) for ( i &lt; N )</td>
</tr>
</tbody>
</table>

Note: \( u_i(U_i, U_{-i}) = b_{i,i}U_i + \sum_{j \neq i} b_{i,j}U_j \).

\( b_{i,i} > 0 \) obtains for \( s < 1 \). Notice that when \( s \in (\frac{1}{2}, 1) \) the indirect effects increase geometrically in \( k - i \), whereas they decrease when \( s < \frac{1}{2} \). The influence from the primary utilities of more affluent agents alternates in sign: \( \frac{\partial u_i}{\partial U_k} < 0 \) for \( k - i > 0 \) odd and \( \frac{\partial u_i}{\partial U_{i+j}} > 0 \) for \( k - i > 0 \) even.

Agent \( i \) directly compares herself with \( i + 1 \) and thus made directly worse off with an increase in \( U_{i+1} \). However, agent \( i \) is made better off with an increase in \( U_{i+2} \). This is because agent \( i + 1 \) directly suffers by comparing herself with agent \( i + 2 \). Thus, agent \( i \) indirectly benefits from an increase in \( U_{i+2} \) as that hurts agent \( i + 1 \).

This example violates Assumption 2 (as \( W_{i-1,i+1} = 0 < W_{i,i+1} = 1 \)). We have \( u_{N-1} < u_N \) and the ranking is preserved for other agents as well, \( u_i < u_{i+1} \), when \( s < \frac{1}{2} \Delta_i \) where \( \Delta_i = \frac{U_{i+2} - U_{i+1}}{U_{i+1} - U_i} \). For equal increments (\( U_{i+1} - U_i \) constant), we have \( \Delta_i = 1 \) for \( i = 1 \ldots N - 2 \) and the ranking is preserved when \( s < \frac{1}{3} \).